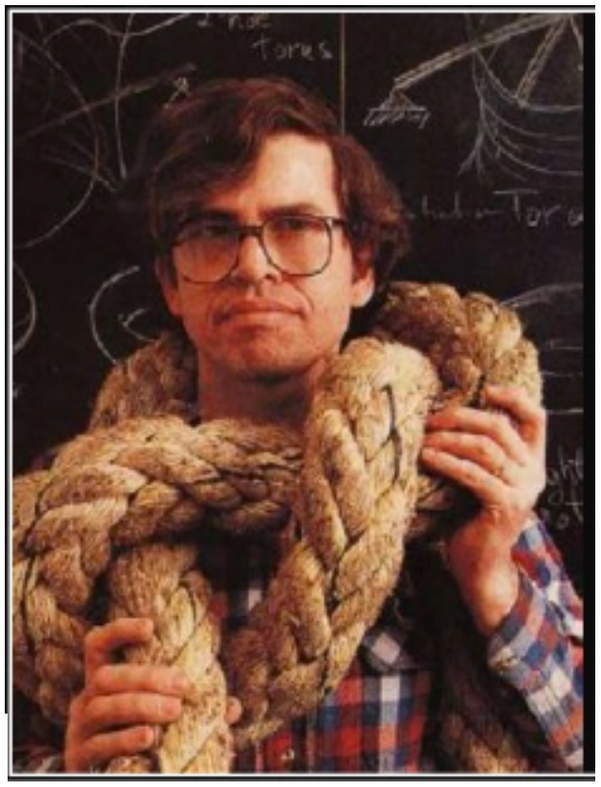
Thurston's last paper

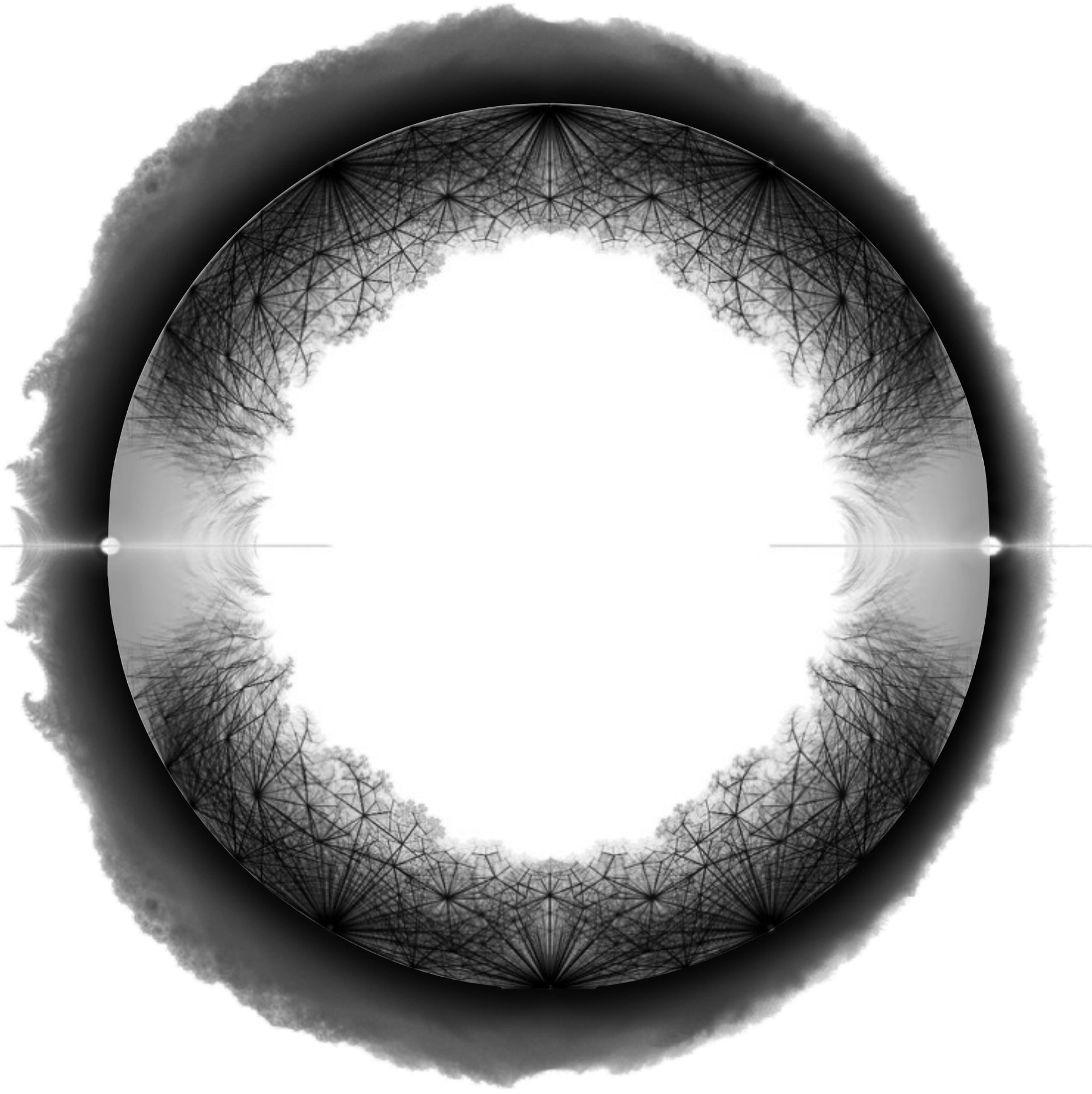
ENTROPY IN DIMENSION ONE

arxiv.org/abs/1402.2008



Bill Thurston

(1946 - 2012)



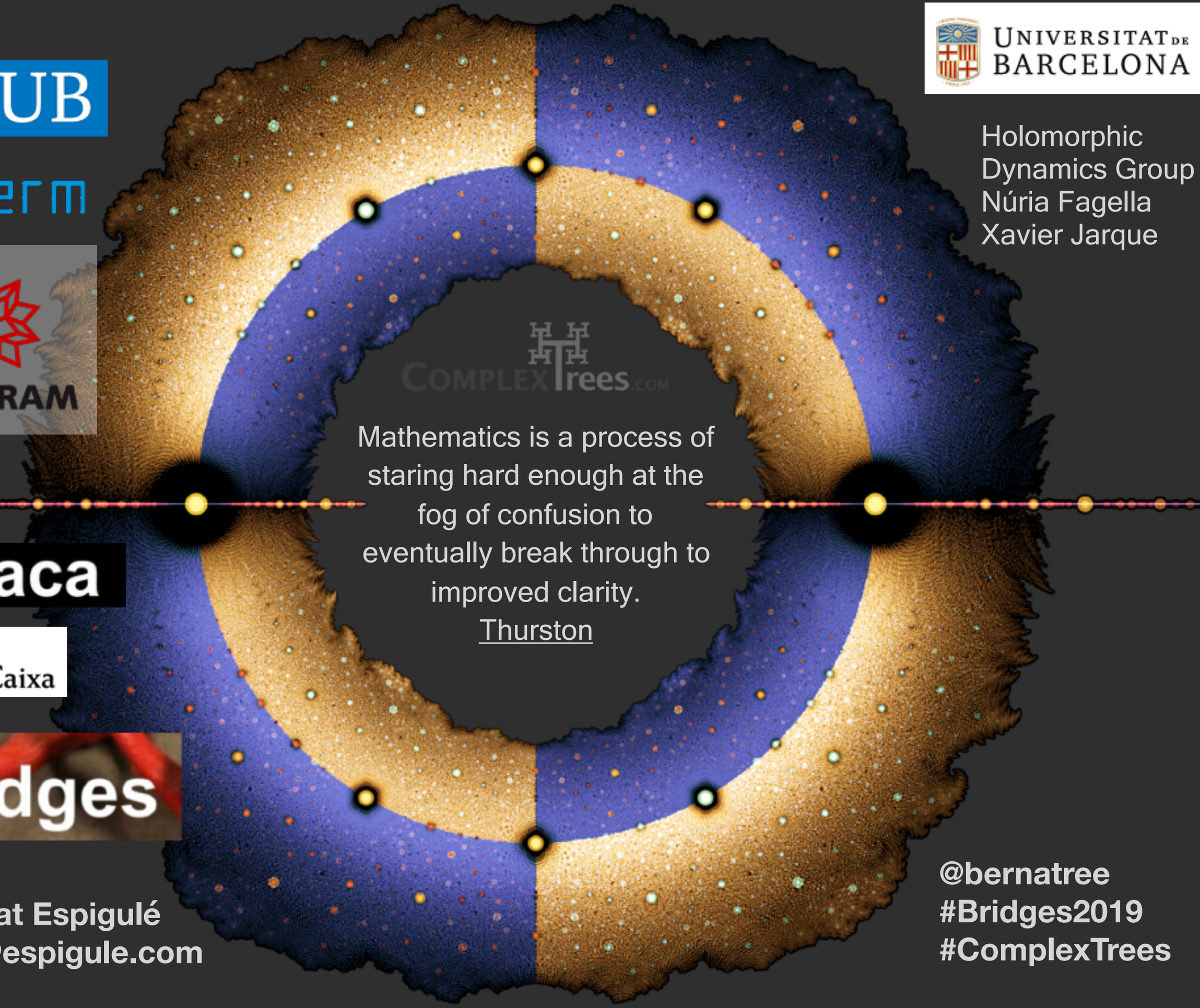
IMUB

icerm

WOLFRAM

UNIVERSITAT DE BARCELONA

Holomorphic Dynamics Group
Núria Fagella
Xavier Jarque



Mathematics is a process of staring hard enough at the fog of confusion to eventually break through to improved clarity.
Thurston

mmaca

"la Caixa"
CosmoCaixa

Bridges

Bernat Espigulé
bernat@espigule.com

@bernatree
#Bridges2019
#ComplexTrees

Geodesics

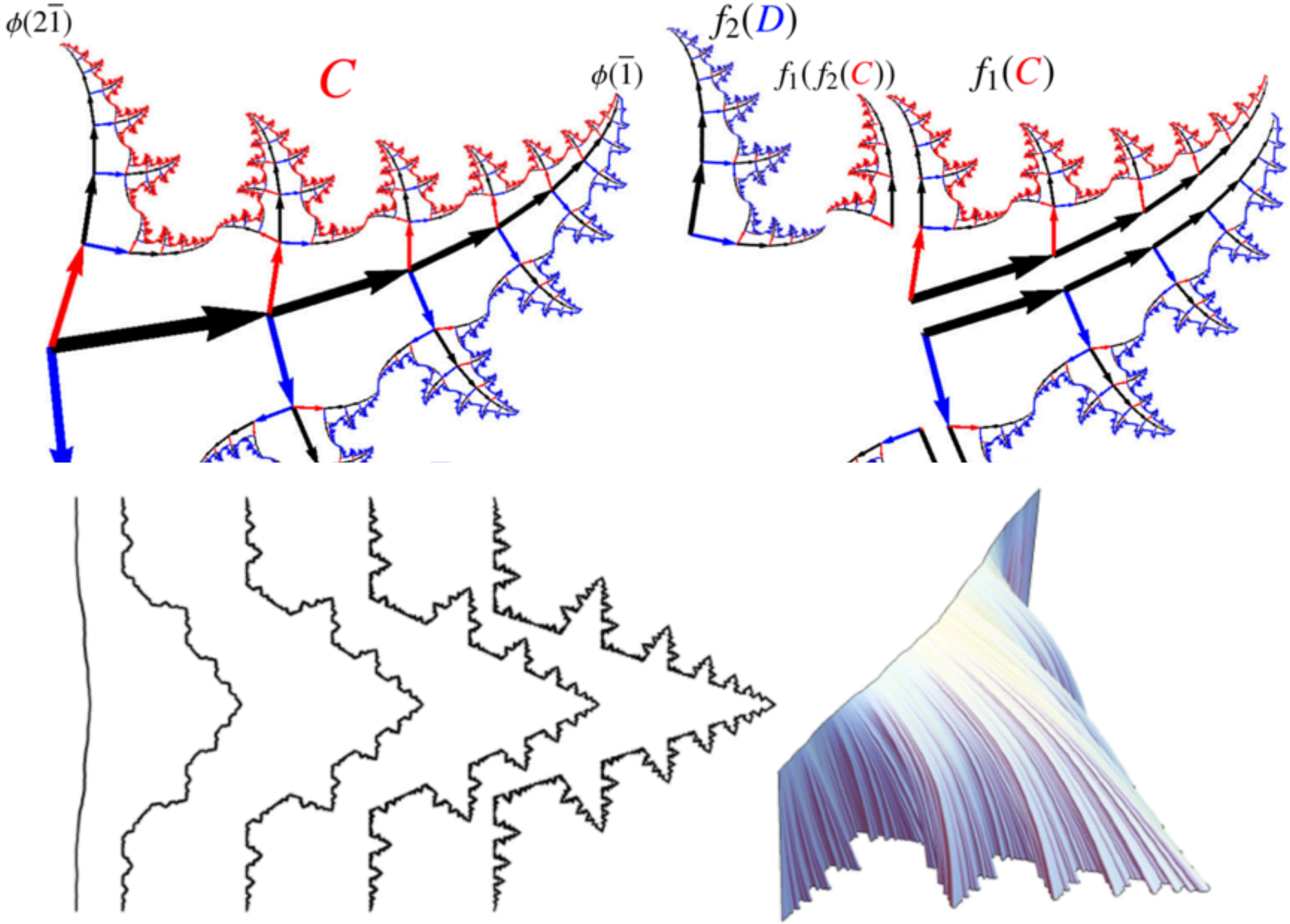
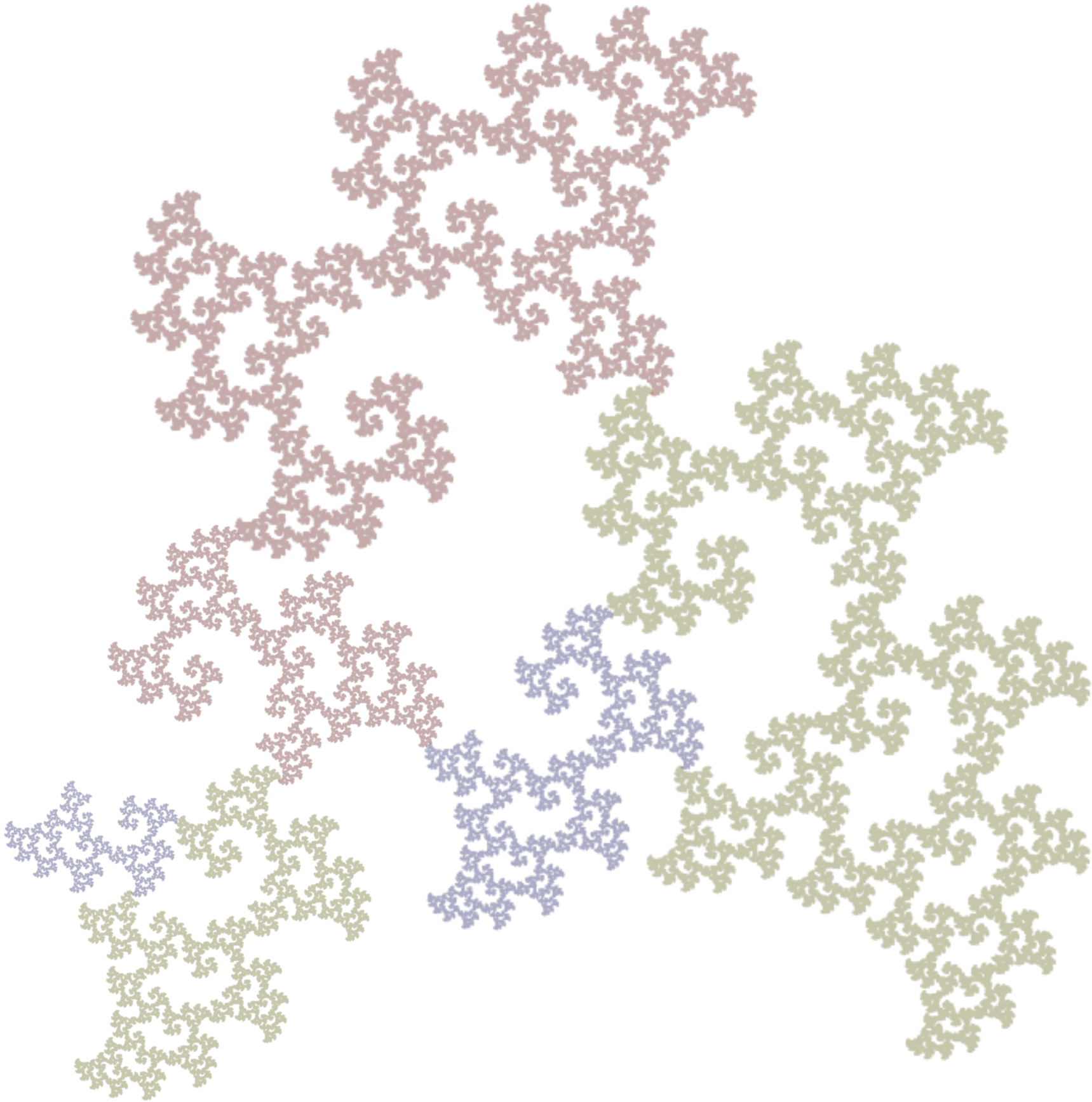
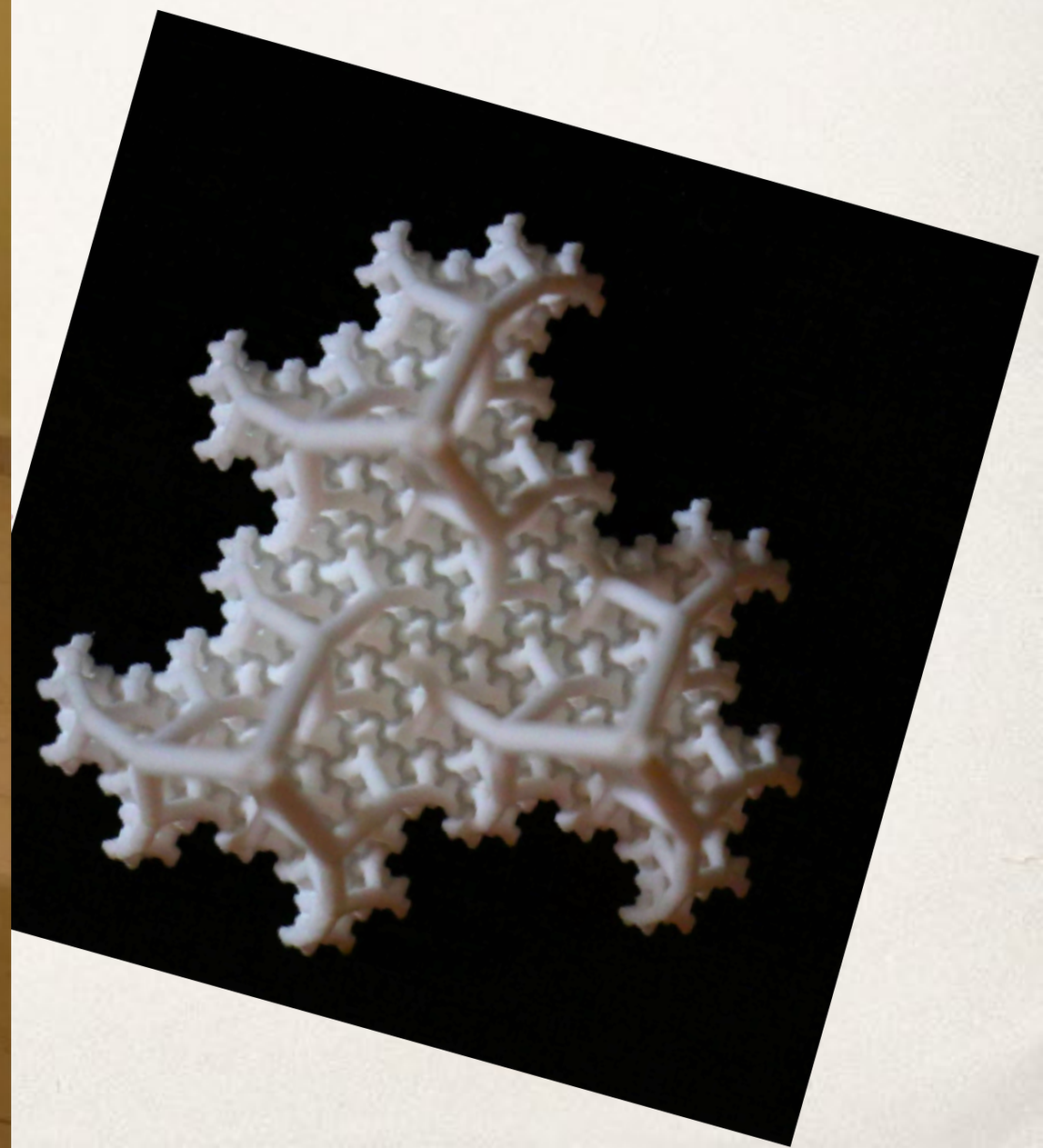
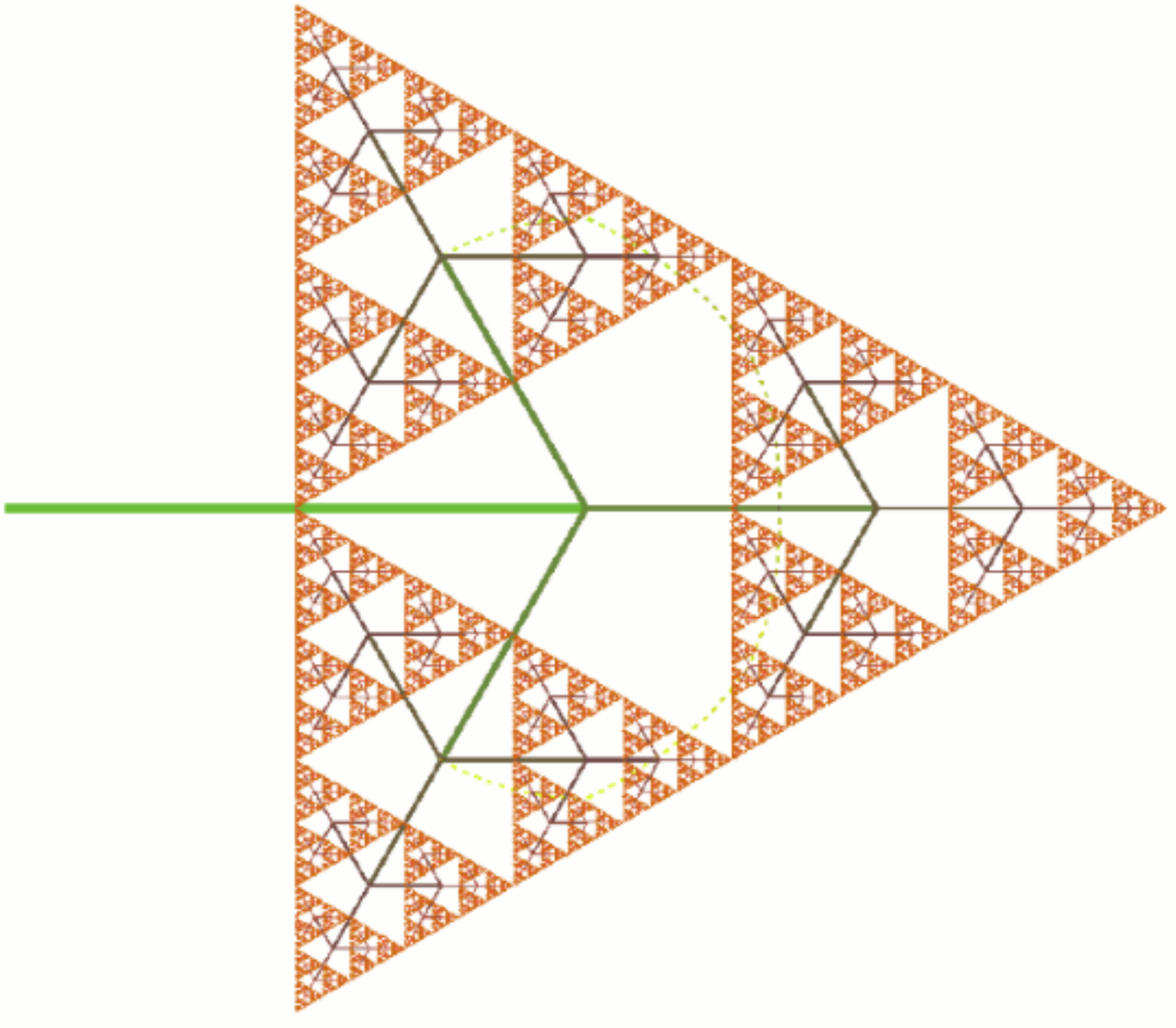


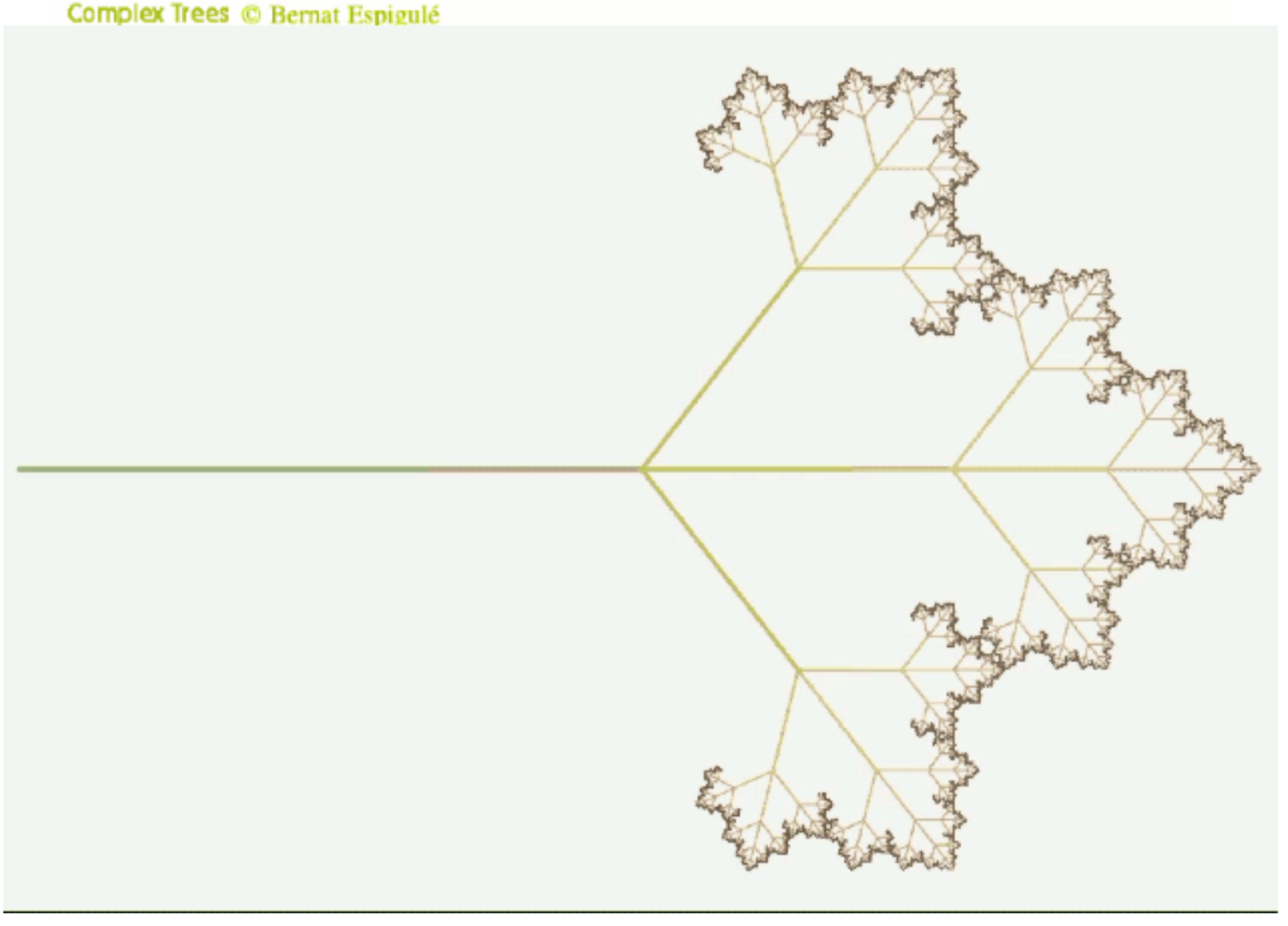
Figure 8: *Shortest paths $C \cup D \subset F_A(z)$ for $z \in (1-\sqrt{2}, 1)$ with the 3D surface generated by them.*

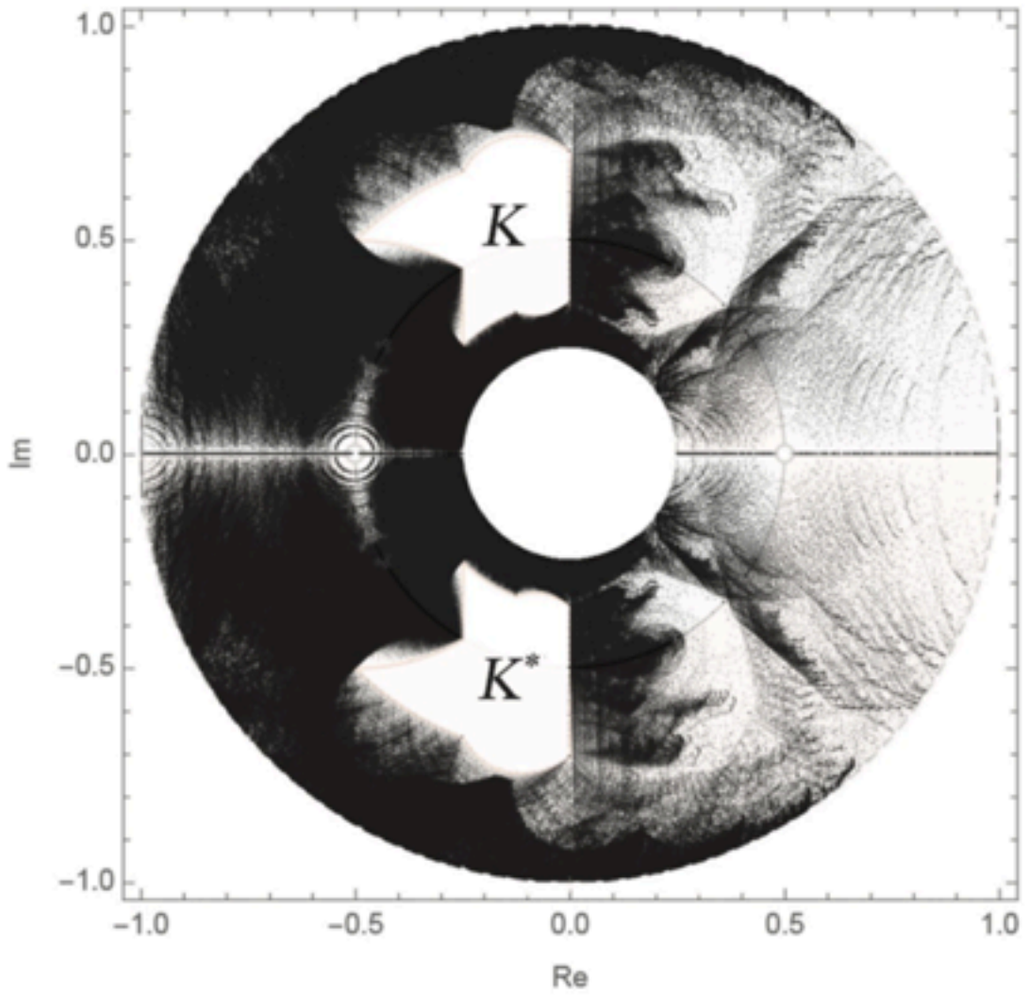


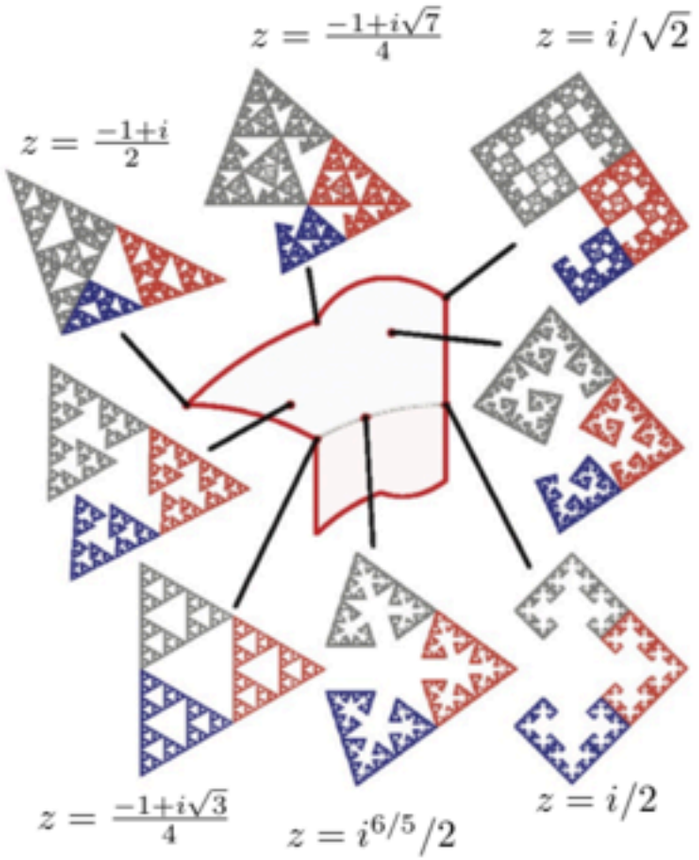




Complex Trees © Bernat Espigulé




1.0
0.5
0.0
−0.5
−1.0
−1.0
−0.5
0.0
0.5
1.0
Re
Im
K
K*


$z = \frac{-1+i\sqrt{7}}{4}$
$z = i/\sqrt{2}$
$z = \frac{-1+i}{2}$
$z = \frac{-1+i\sqrt{3}}{4}$
$z = i^{6/5}/2$
$z = i/2$

Key idea: “tip-to-tip gluing”

$$\phi(13\overline{2}) = 1 + c_1 + c_1 c_3 \sum_{k=0}^{\infty} c_2^k = 1 + c_1 + \frac{c_1 c_3}{1 - c_2}$$

$$\phi(21\overline{2}) = 1 + c_2 + \frac{c_2 c_1}{1 - c_2}$$

$$\phi(23\overline{2}) = 1 + c_2 + \frac{c_2 c_3}{1 - c_2}$$

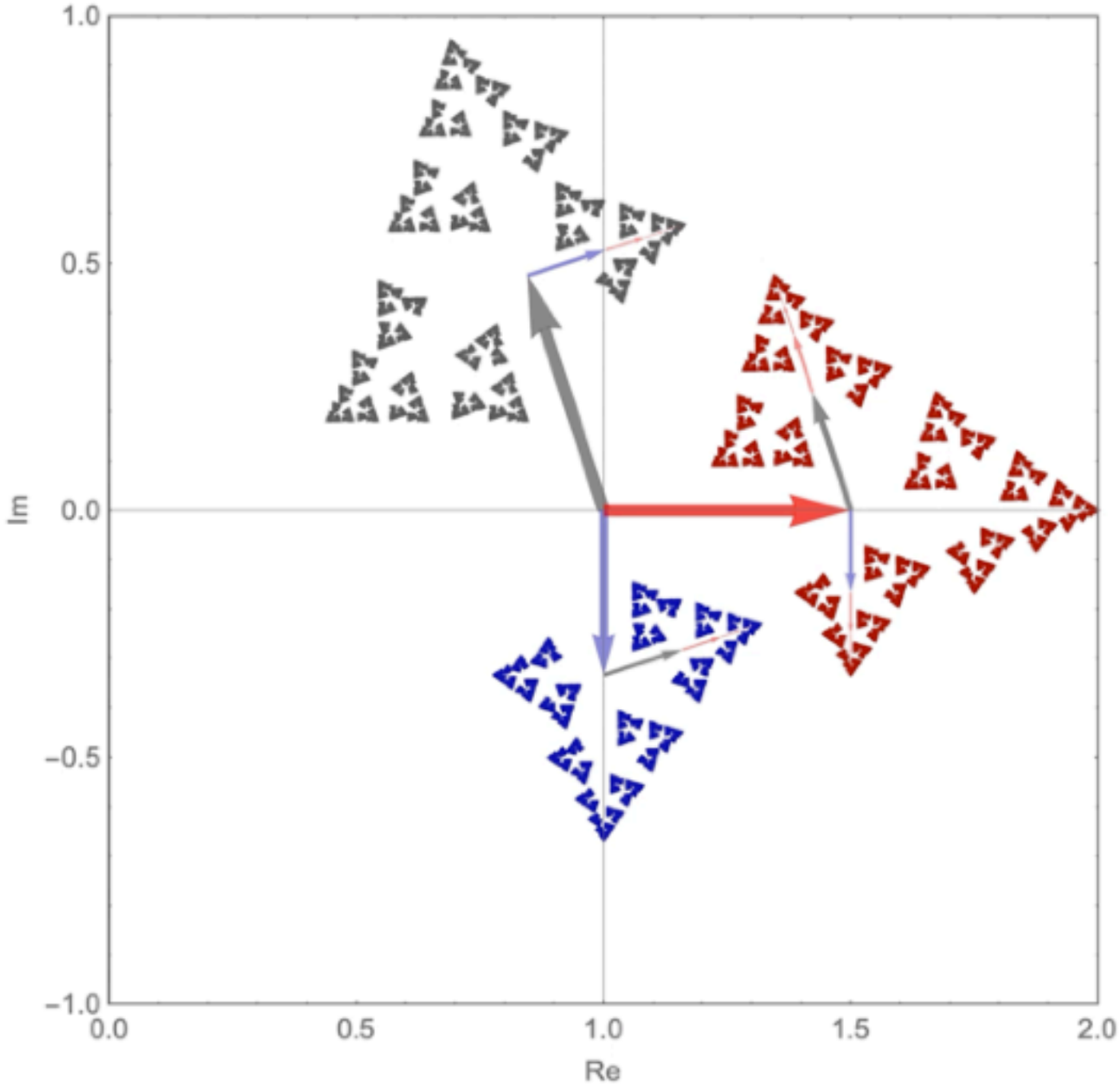
$$\phi(31\overline{2}) = 1 + c_3 + \frac{c_3 c_1}{1 - c_2}$$

$$\begin{cases} \phi(13\overline{2}) = \phi(21\overline{2}) \\ \phi(23\overline{2}) = \phi(31\overline{2}) \end{cases} \Longrightarrow \begin{cases} c_2 = 1/2 \\ c_3 = 1/4 c_1 \end{cases}$$

$$c_1(z) := z, \quad c_2(z) := 1/2, \quad c_3(z) := 1/4z$$

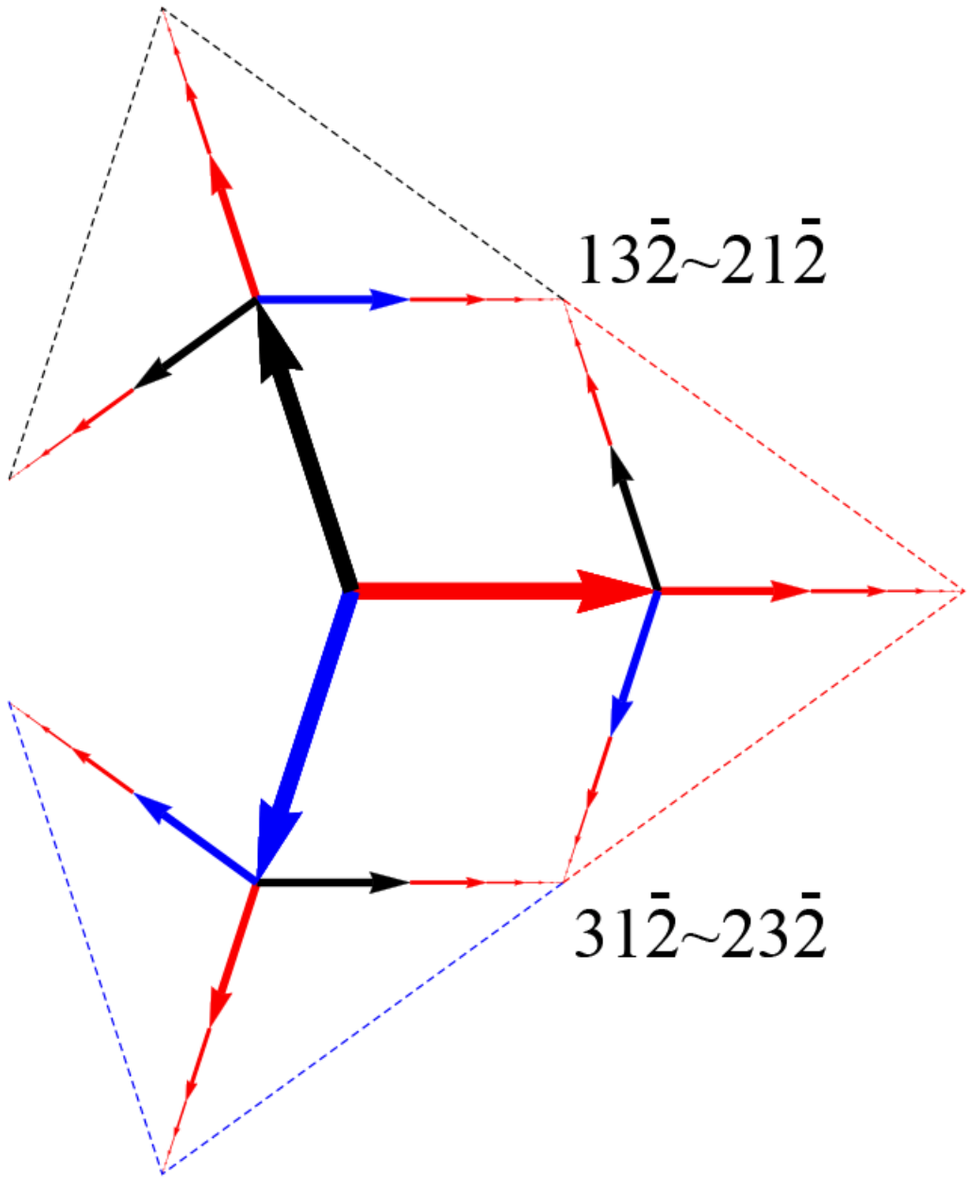
$$T_A(z) = T\{c_1(z), c_2(z), c_3(z)\} = T\{z, 1/2, 1/4z\}$$

$$T_A(i^{6/5}/2) = T\{\frac{i^{6/5}}{2}, \frac{1}{2}, -\frac{i^{4/5}}{2}\}$$



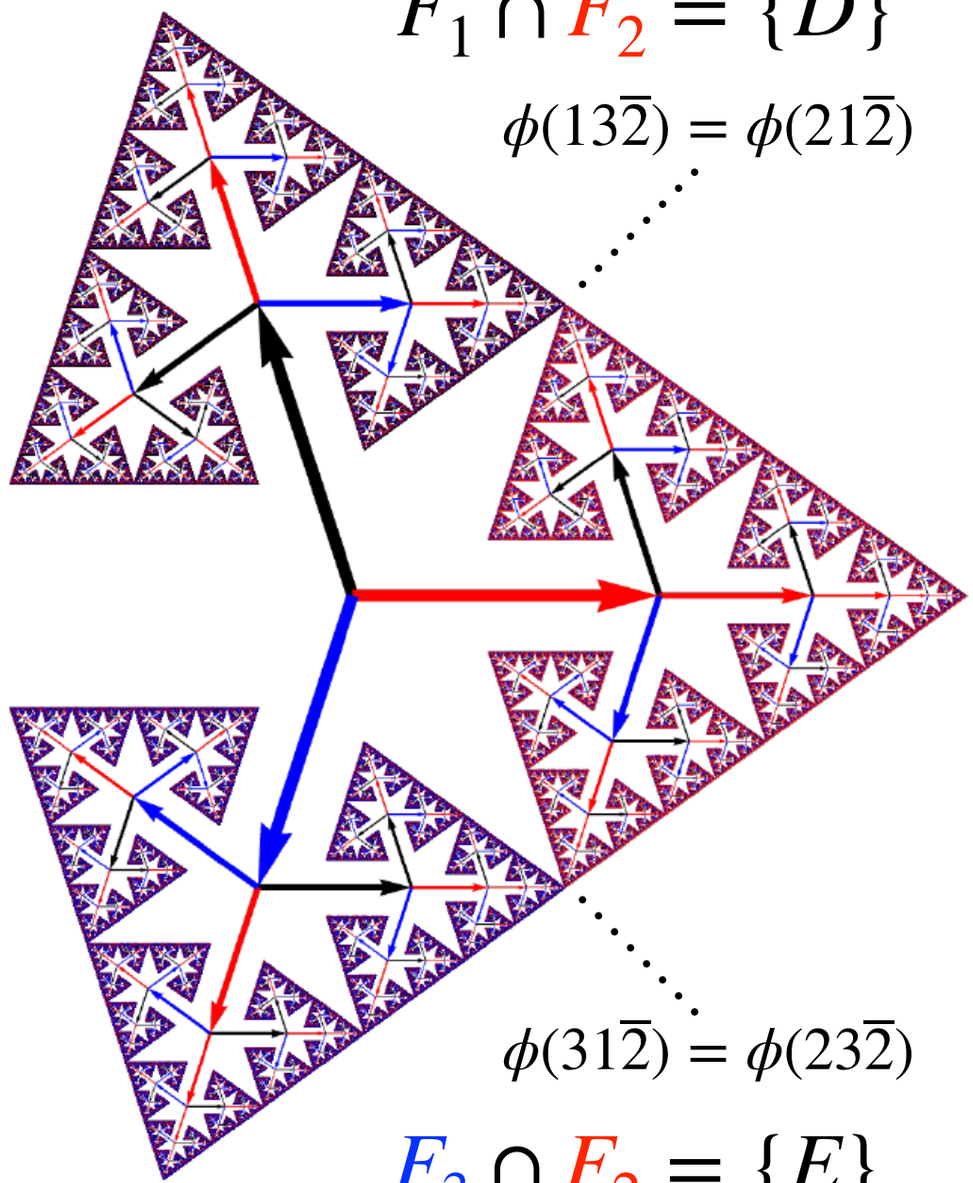
Topological set

$$Q_A(i^{6/5}/2) = \{13\bar{2} \sim 21\bar{2},\ 31\bar{2} \sim 23\bar{2}\}$$



$$T_A(i^{6/5}/2) = T\{\frac{i^{6/5}}{2}, \frac{1}{2}, -\frac{i^{4/5}}{2}\}$$

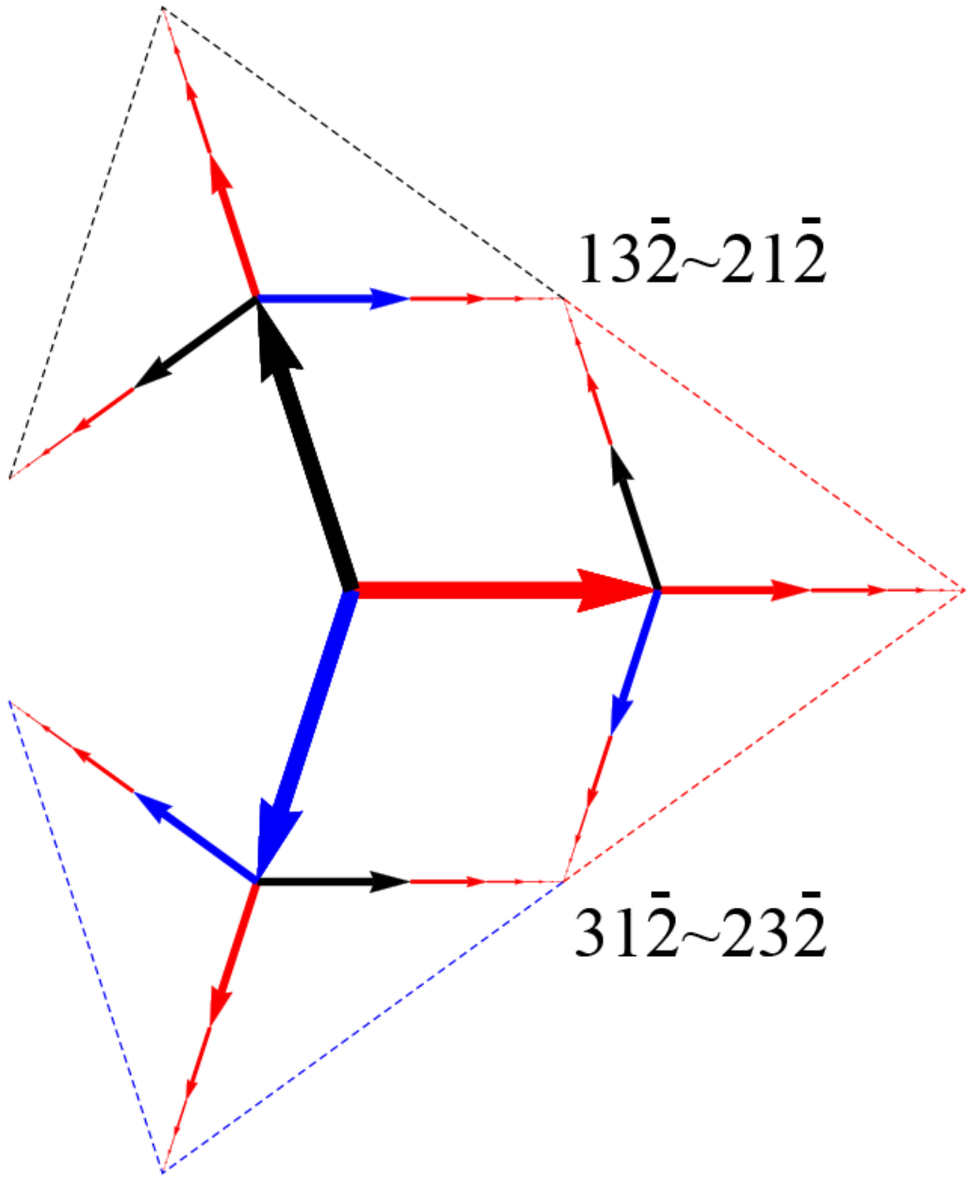
$$F_1 \cap F_2 = \{D\}$$



$$F_3 \cap F_2 = \{E\}$$

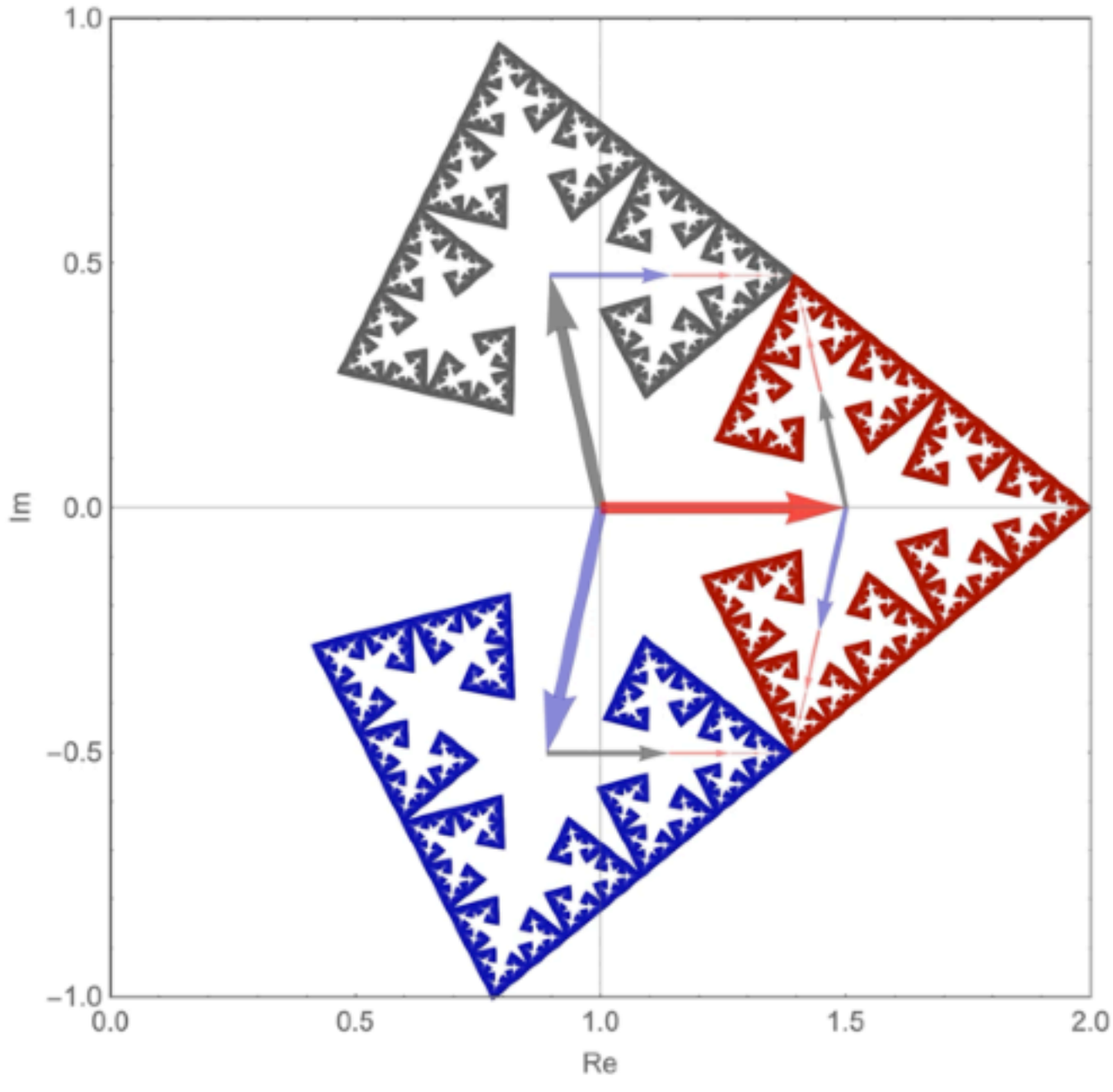
Topological set

$$Q_A(i^{6/5}/2+\epsilon) = \{13\bar{2} \sim 21\bar{2},\ 31\bar{2} \sim 23\bar{2}\}$$



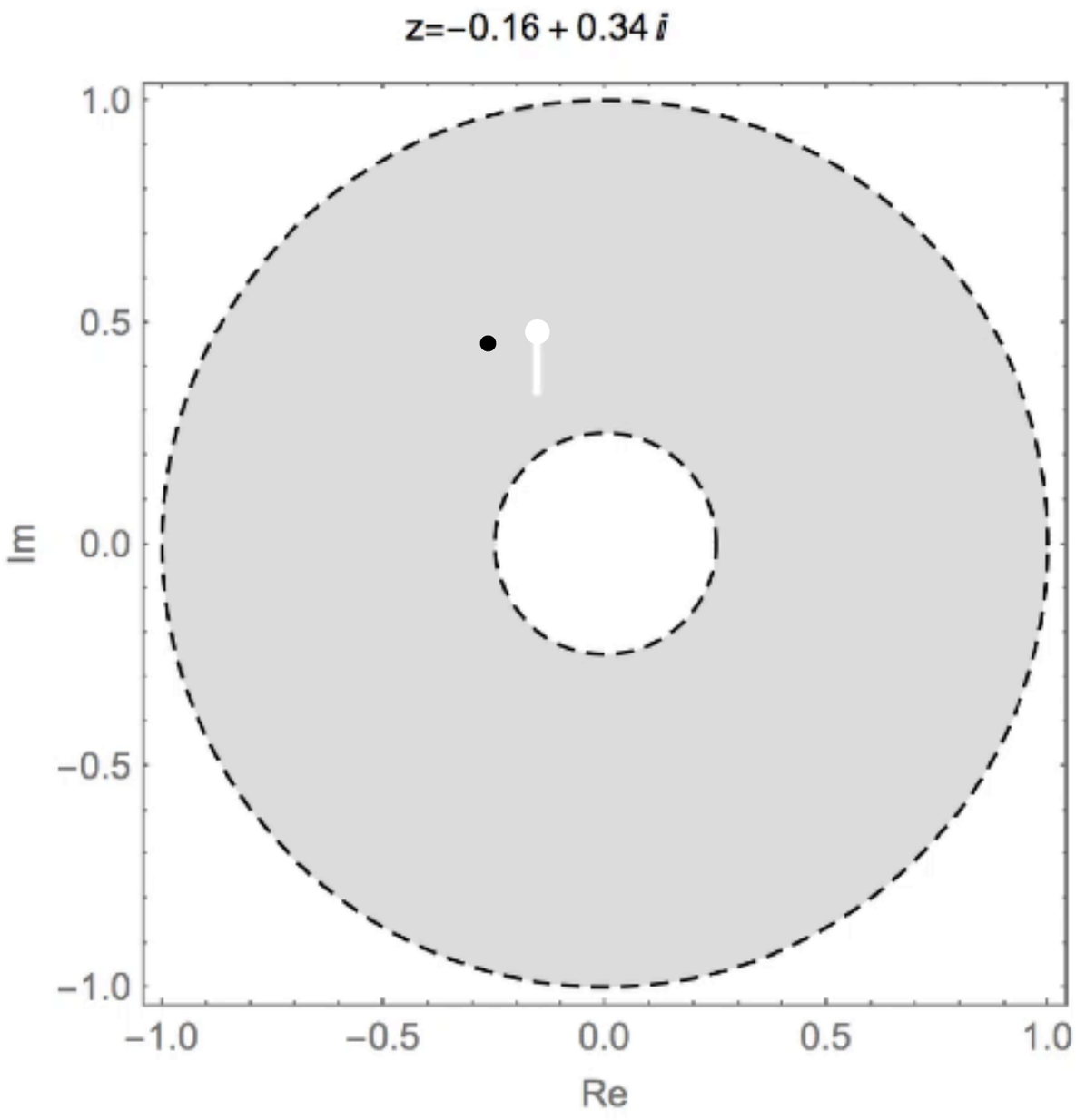
Structurally stable tree

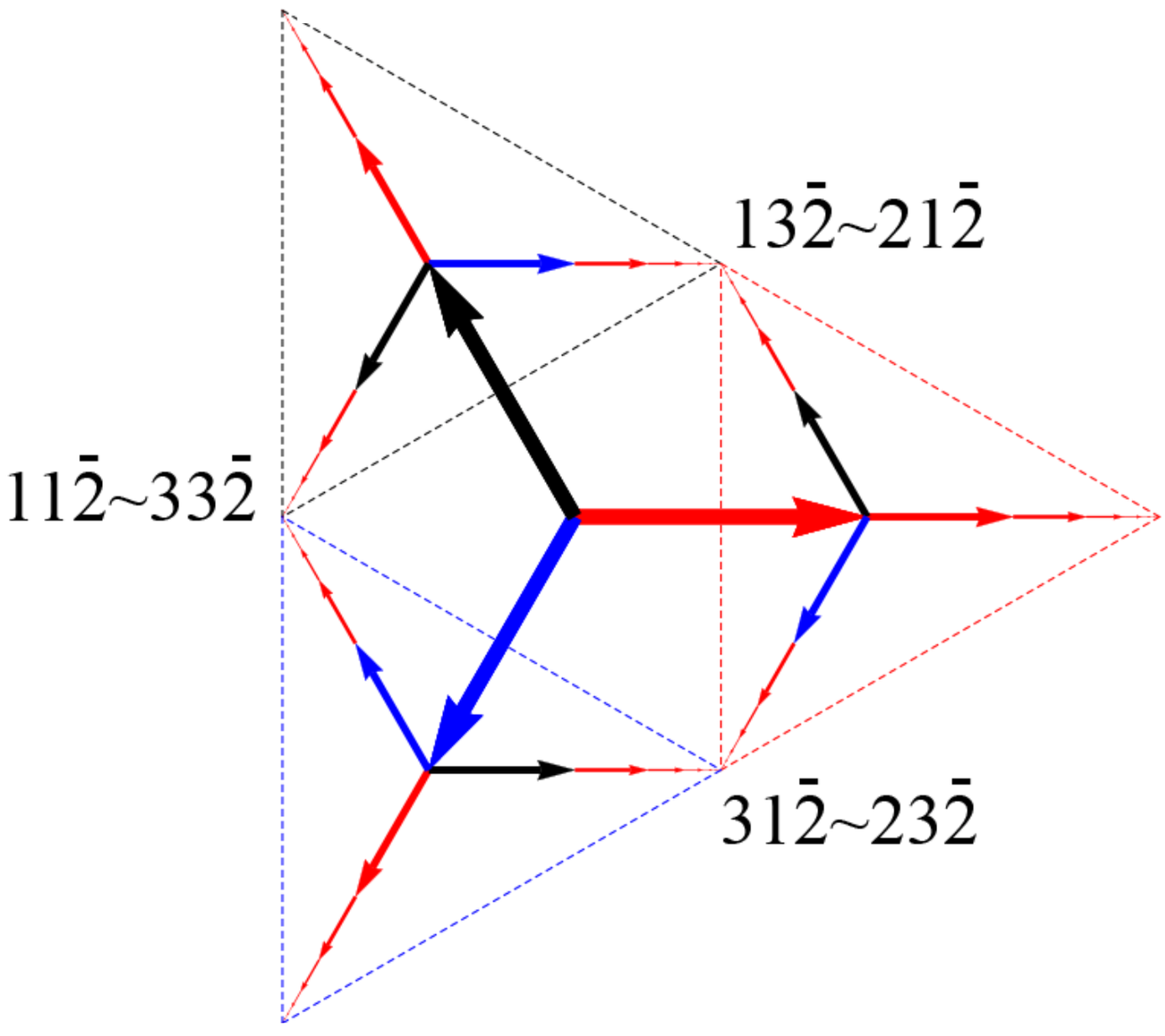
$$T_A(i^{6/5}/2+\epsilon) = T\{i^{6/5}/2+\epsilon, \frac{1}{2}, c_3(i^{6/5}/2+\epsilon)\}$$



A family of tipset connected trees $T_A(z) := T\{z, \frac{1}{2}, \frac{1}{4z}\}$

$$Q_A(-1/4 + i\sqrt{3}/4) = \{13\bar{2} \sim 21\bar{2},\ 31\bar{2} \sim 23\bar{2},\ 11\bar{2} \sim 33\bar{2}\}$$

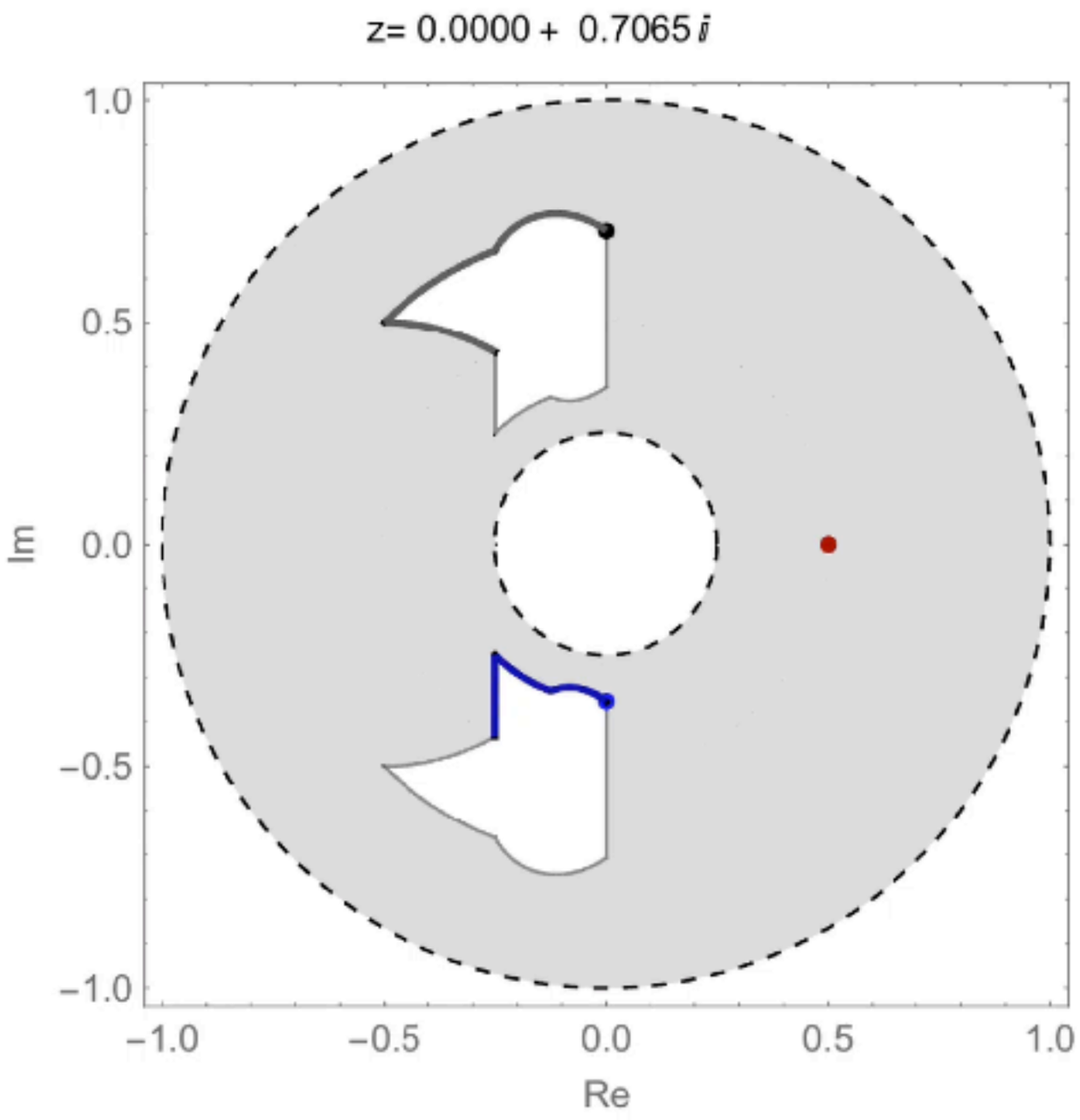


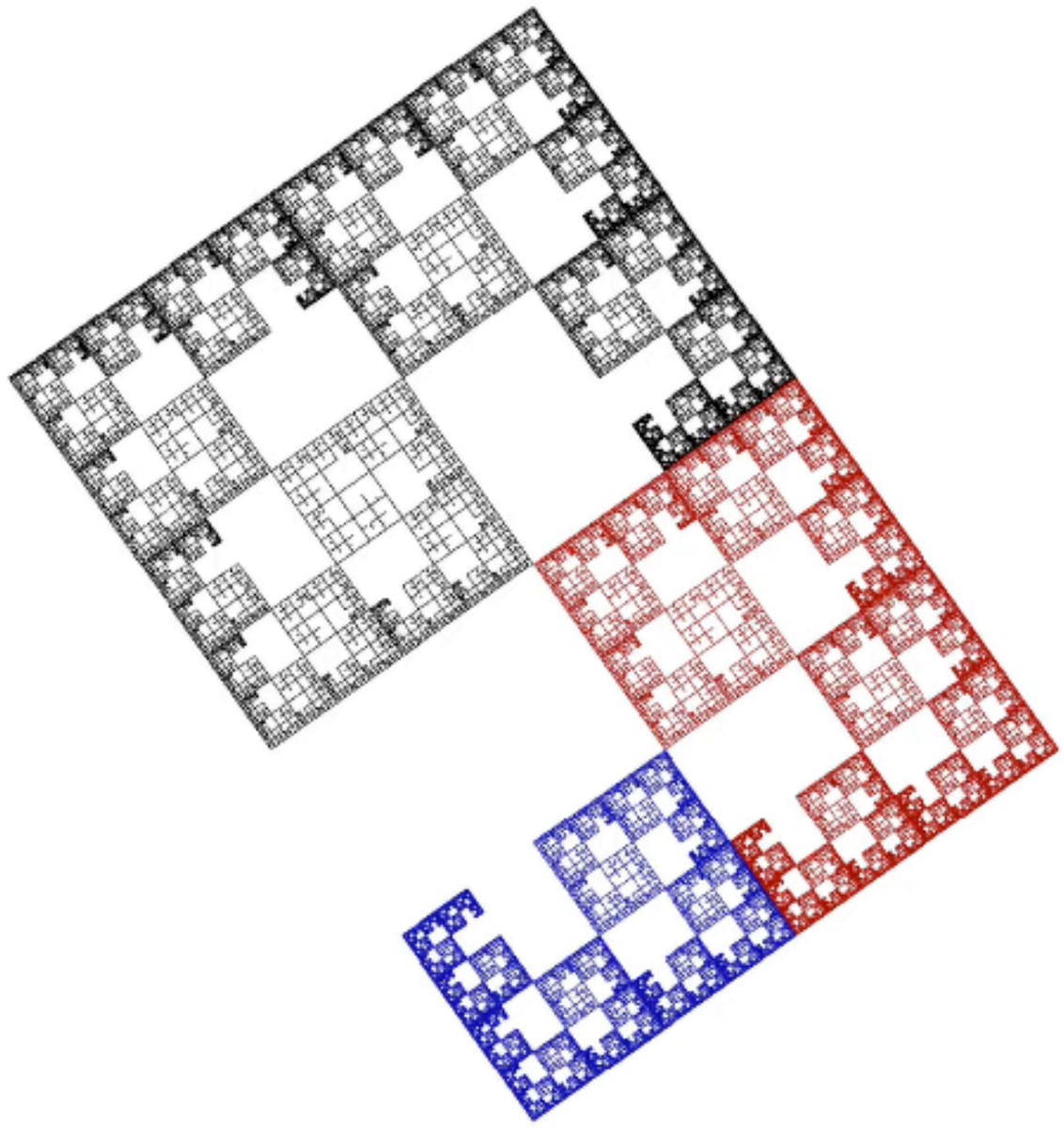


$$\mathscr{R} := \{z \in \mathbb{C} : 1/4 < |z| < 1\}$$

A family of tipset connected trees $T_A(z) := T\{z, \frac{1}{2}, \frac{1}{4z}\}$

$$Q_A(z) \supset \{13\overline{2} \sim 21\overline{2},\ 31\overline{2} \sim 23\overline{2}\}$$





$$\mathscr{R} := \{z \in \mathbb{C} : 1/4 < |z| < 1\}$$

All the branches of a tree at every stage of its height when put together are equal in thickness to the trunk.

All the branches of a water [course] at every stage of its course, if they are of equal rapidity, are equal to the body of the main stream.

– Leonardo da Vinci 1452-1519