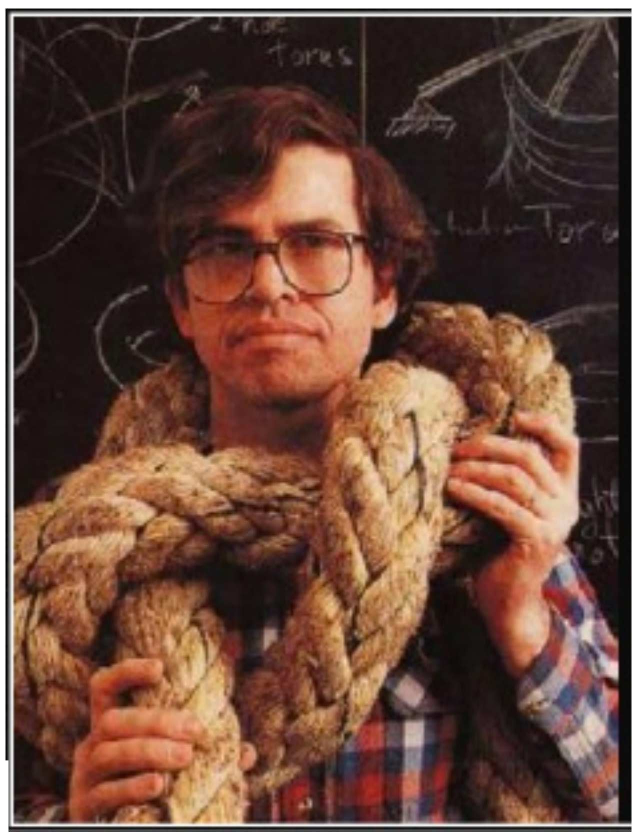
Thurston’s last paper

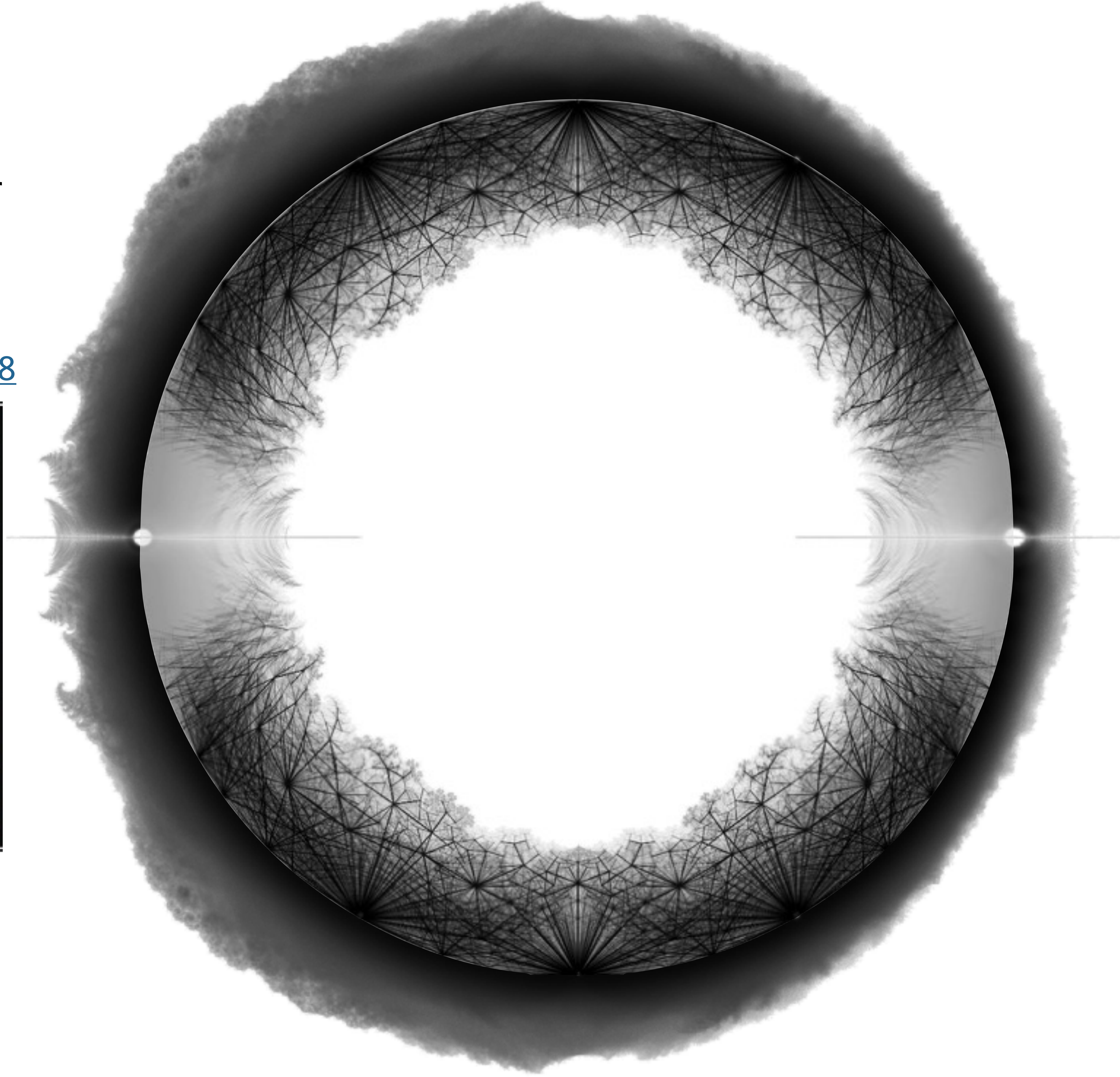
ENTROPY IN DIMENSION ONE

arxiv.org/abs/1402.2008



Bill Thurston

(1946 – 2012)



https://math.univ-angers.fr/~tanlei/papers/Barcelona-entropy.pdf

Giulio Tiozzo

Assistant Professor of Mathematics
University of Toronto

My field of research is dynamical systems and ergodic theory. In particular:

- Entropy and Hausdorff dimension in the quadratic family;
- Random walks on groups, especially the mapping class group;
- Ergodic theory of generalized continued fraction transformations.

For a more detailed description, see my research statement.
I got my Ph.D. from Harvard in 2013, under the supervision of C.T. McMullen.
In my thesis, I develop a connection between families of continued fractions and the combinatorics of the Mandelbrot set, with applications to entropy.

Entropy on Hubbard trees

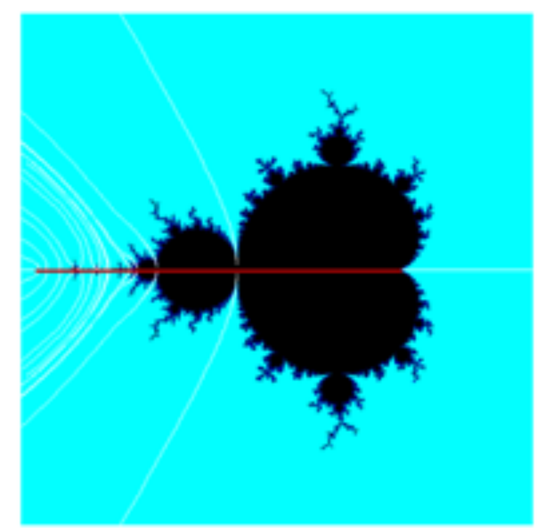
Tan Lei 譚蕾, Université d'Angers

(most figures are produced by W. Thurston and G. Tiozzo)

Barcelona, June 2013

Tiozzo's section theorem ('2012)

Set $P_c := \{\theta \in S^1 :$ the parameter θ-ray lands on $[c, \frac{1}{4}[\}$.



Then, $\dfrac{\text{core-entropy}(f_c)}{\log 2} = \text{H.dim } P_c$.

The proof uses techniques coming from α-continued fractions.

THE SHAPE OF THURSTON'S MASTER TEAPOT

HARRISON BRAY, DIANA DAVIS, KATHRYN LINDSEY AND CHENXI WU

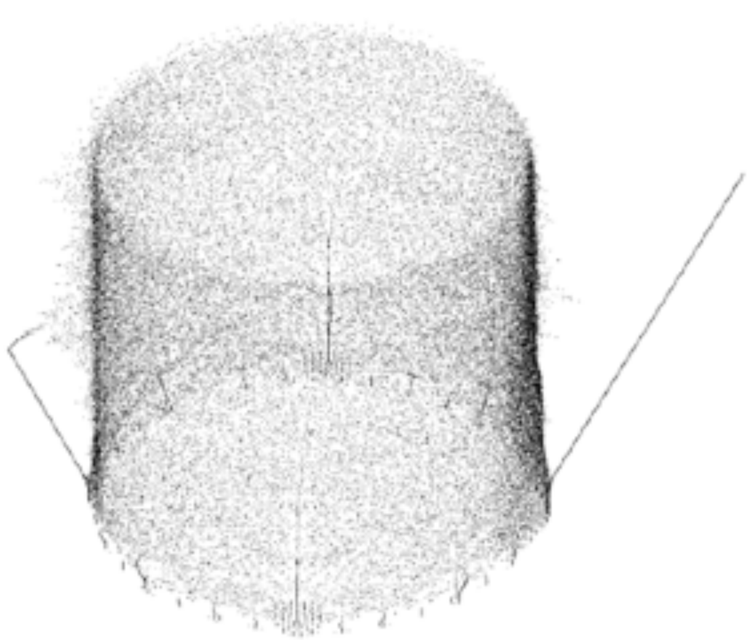
2019

ABSTRACT. We establish basic geometric and topological properties of Thurston's Master Teapot and the Thurston set for superattracting unimodal continuous self-maps of intervals. In particular, the Master Teapot is connected, contains the unit cylinder, and its intersection with a set $\mathbb{D} \times \{c\}$ grows monotonically with c. We show that the Thurston set described above is not equal to the Thurston set for postcritically finite tent maps, and we provide an arithmetic explanation for why certain gaps appear in plots of finite approximations of the Thurston set.

A CHARACTERIZATION OF THURSTON'S MASTER TEAPOT

KATHRYN LINDSEY AND CHENXI WU

ABSTRACT. We prove an explicit characterization of the points in Thurston's Master Teapot. This description can be implemented algorithmically to test whether a point in $\mathbb{C} \times \mathbb{R}$ belongs to the complement of the Master Teapot. As an application, we show that the intersection of the Master Teapot with the unit cylinder is not symmetrical under reflection through the plane that is the product of the imaginary axis of $\mathbb{C}$ and $\mathbb{R}$.



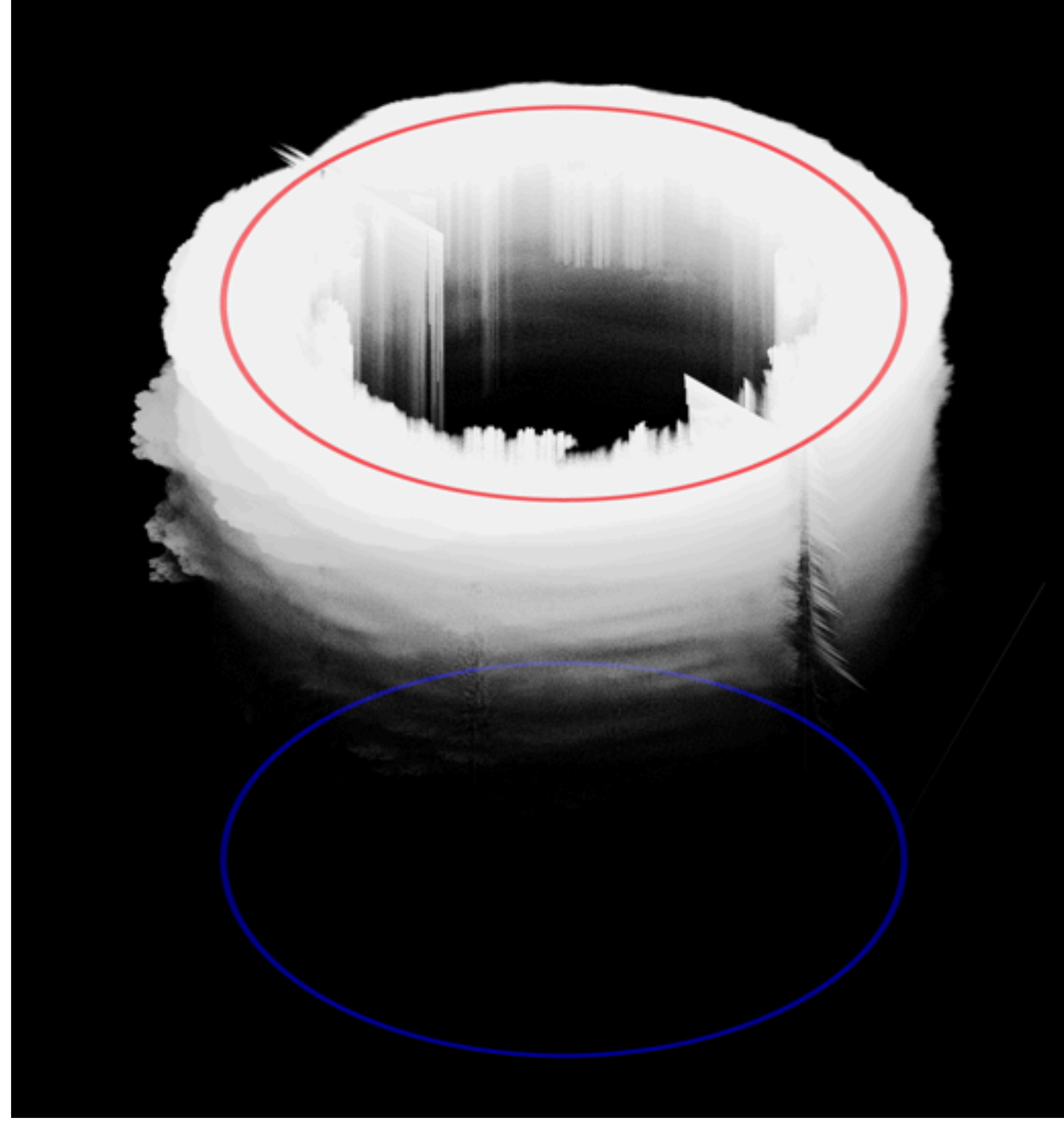
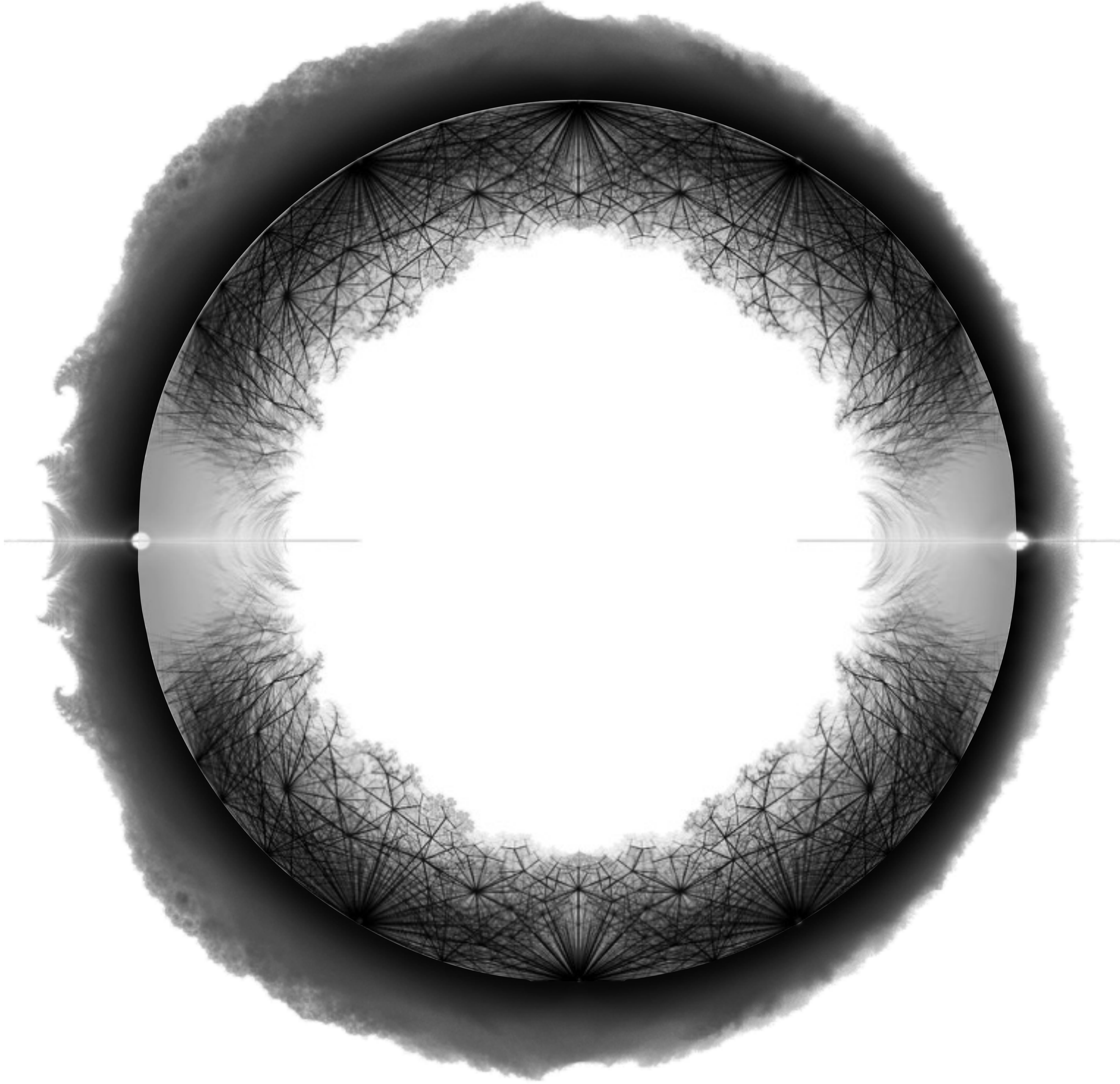
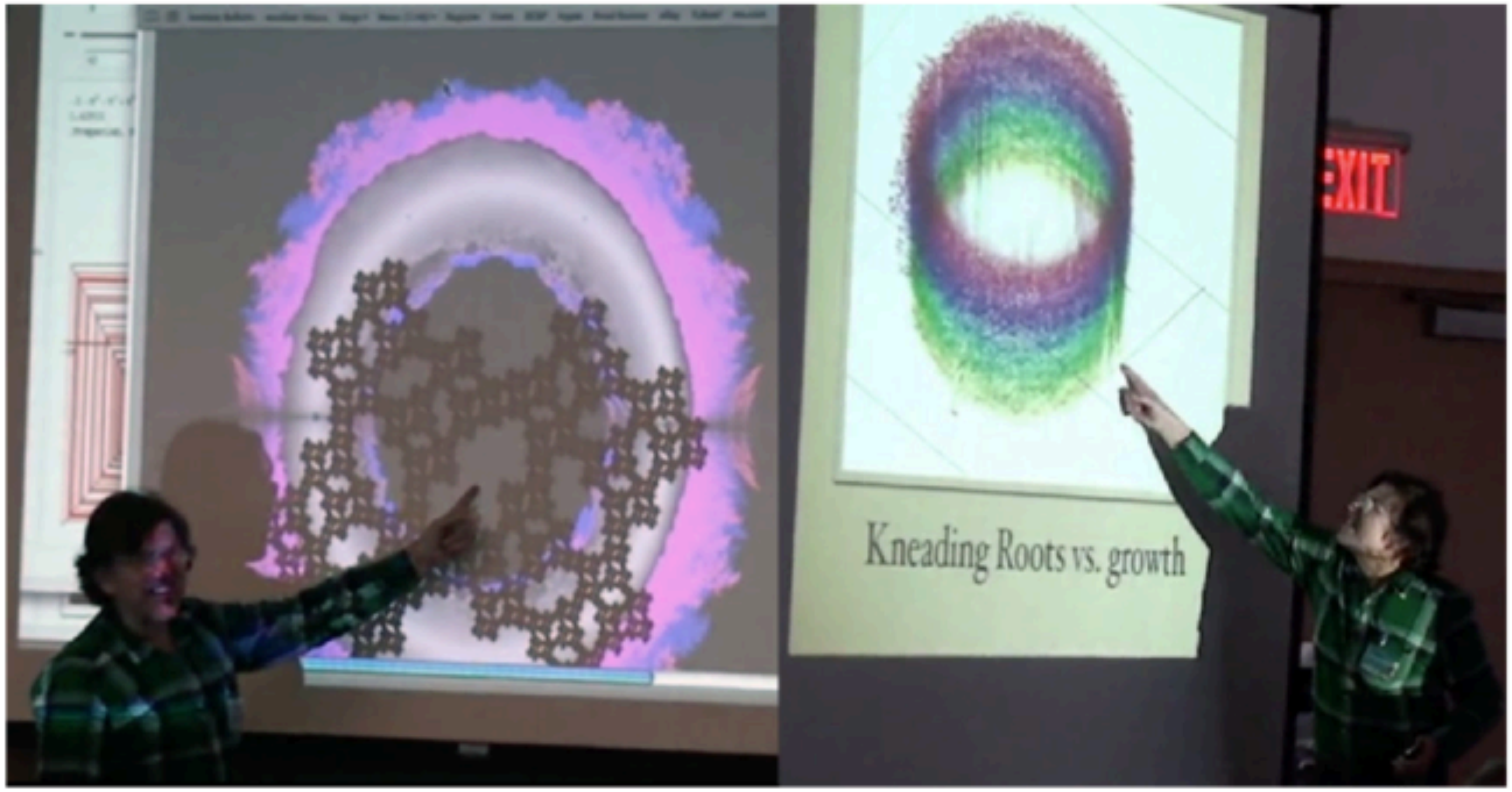
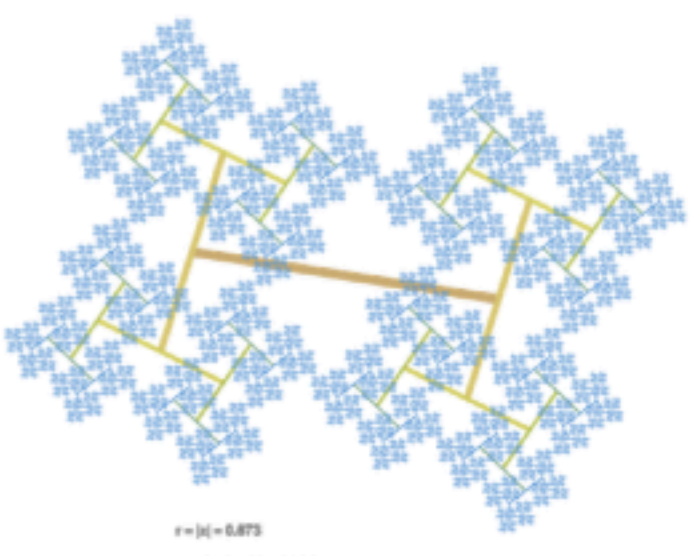


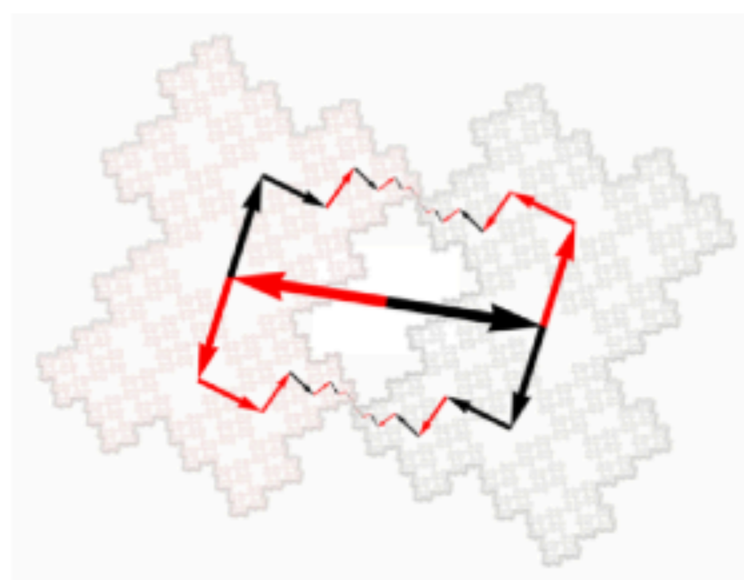
FIGURE 1. An approximation of Thurston's Master Teapot, Υ_2. The horizontal plane is $\mathbb{C}$ and the vertical axis is $\mathbb{R}$. The projection onto $\mathbb{C}$ of the Master Teapot, Υ_2, is the Thurston set, Ω_2. The slice of the teapot at level $z = 1$ is the unit circle (blue); the unit circle is also shown at level $z = 2$ (red). The faint "spout" on the right consists of points the form $(\beta, 0, \beta) \in \mathbb{R}^3 \simeq \mathbb{C} \times \mathbb{R}$.

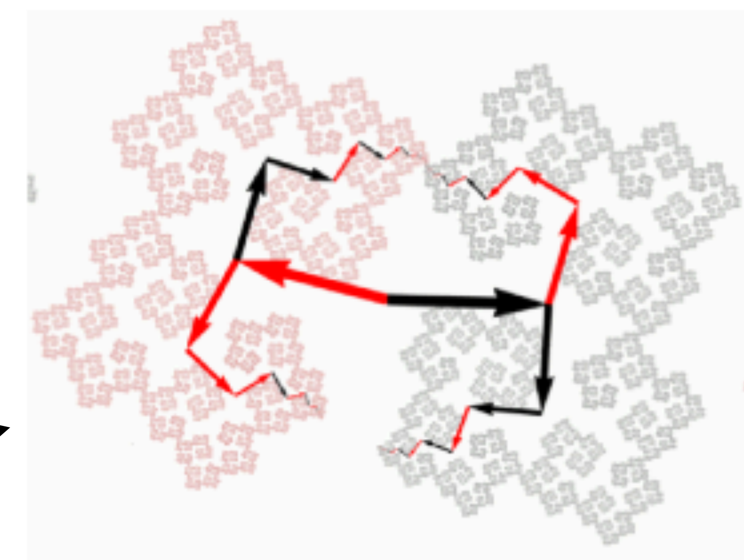


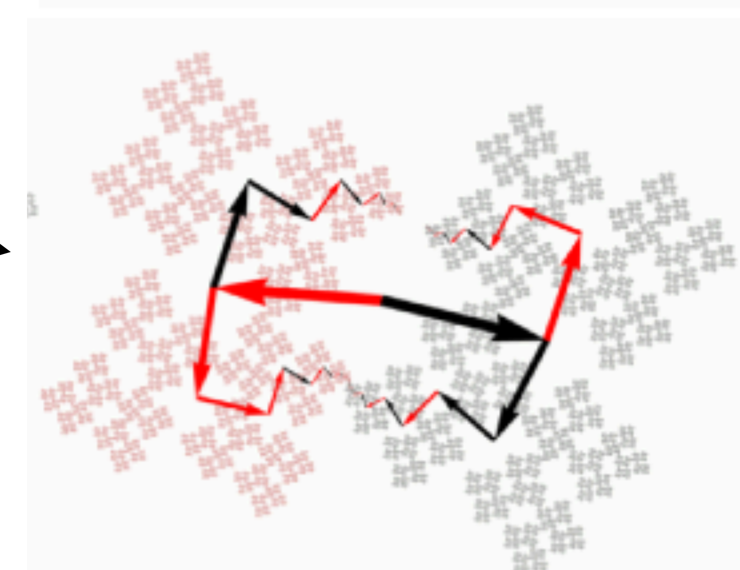


Bill Thurston presenting *M*, *M*_0, and the Thurston Set, Jackfest 2011
[Structure of Entropy: The hidden dimensions]







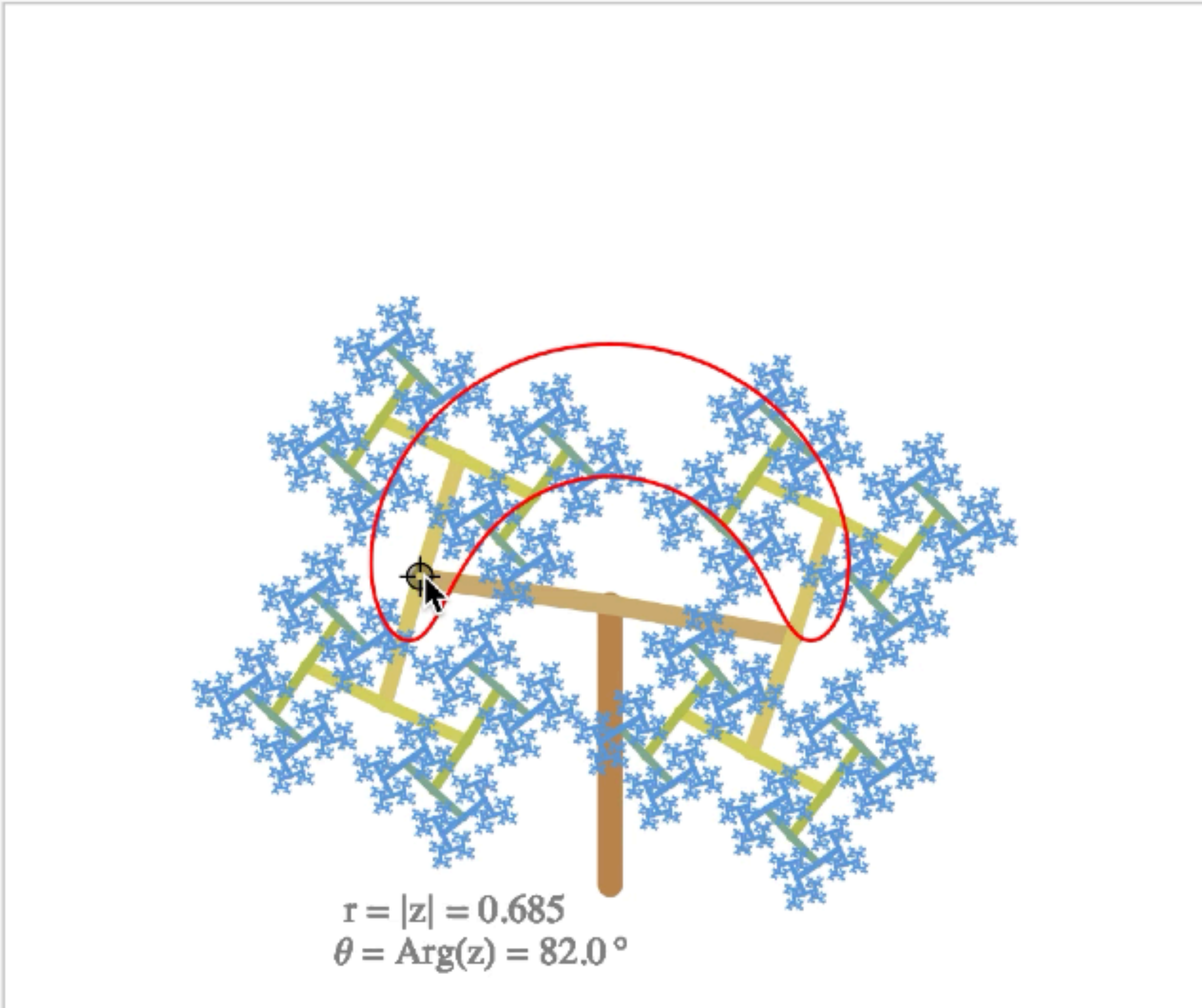


2-gon complex tree with a pair of intersecting points encoded by the following branch path pairs: 122212121…~2112121212... and 11121212..~2221212121… . From these tip-to-tip equivalence relations we can jump to a pair of new families of connected self-similar sets:

Examples_Binary.nb

```
1/(z (2 + z)) /. {z -> pt1[[2]] + I pt1[[1]]}
```

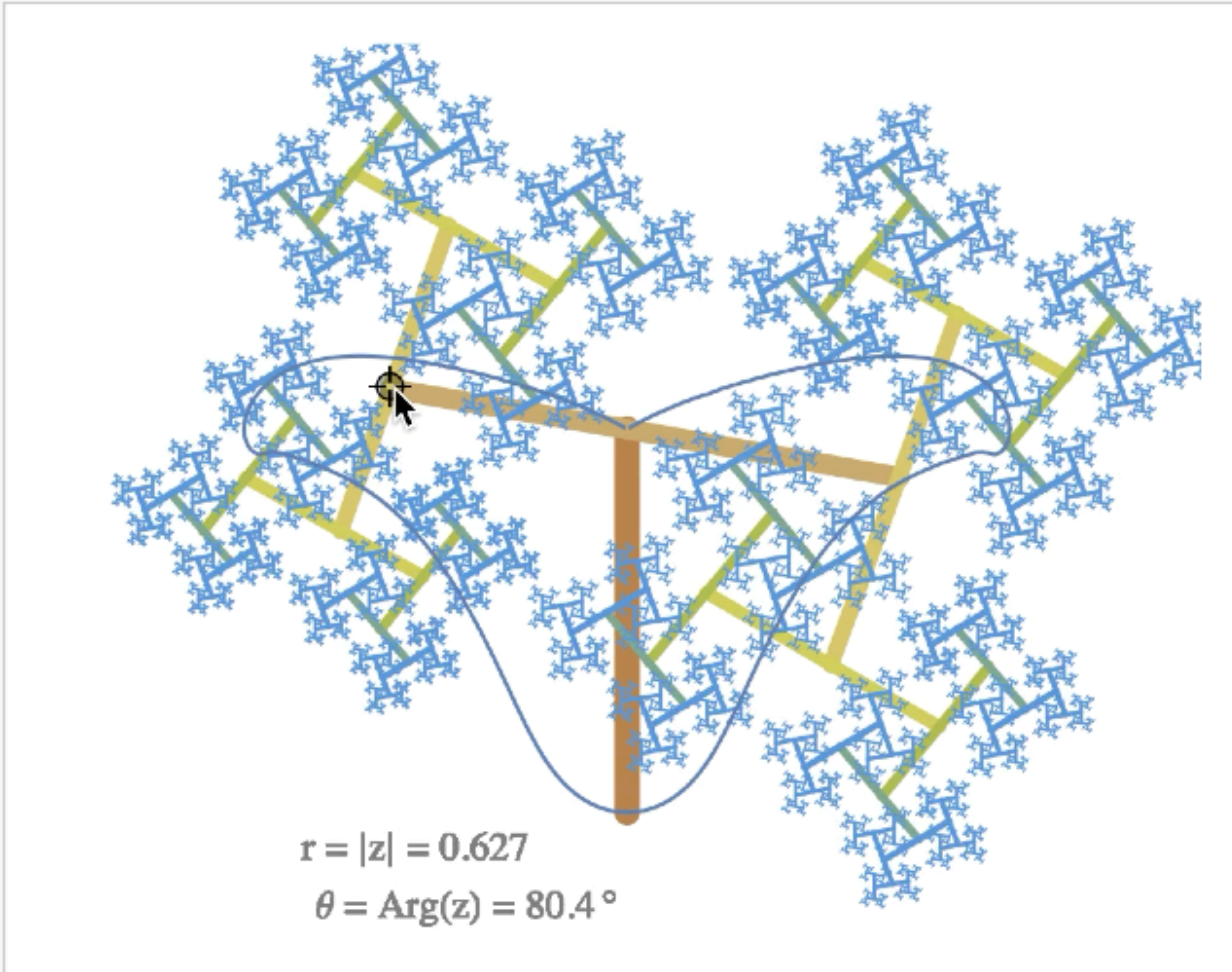
Out[1]=

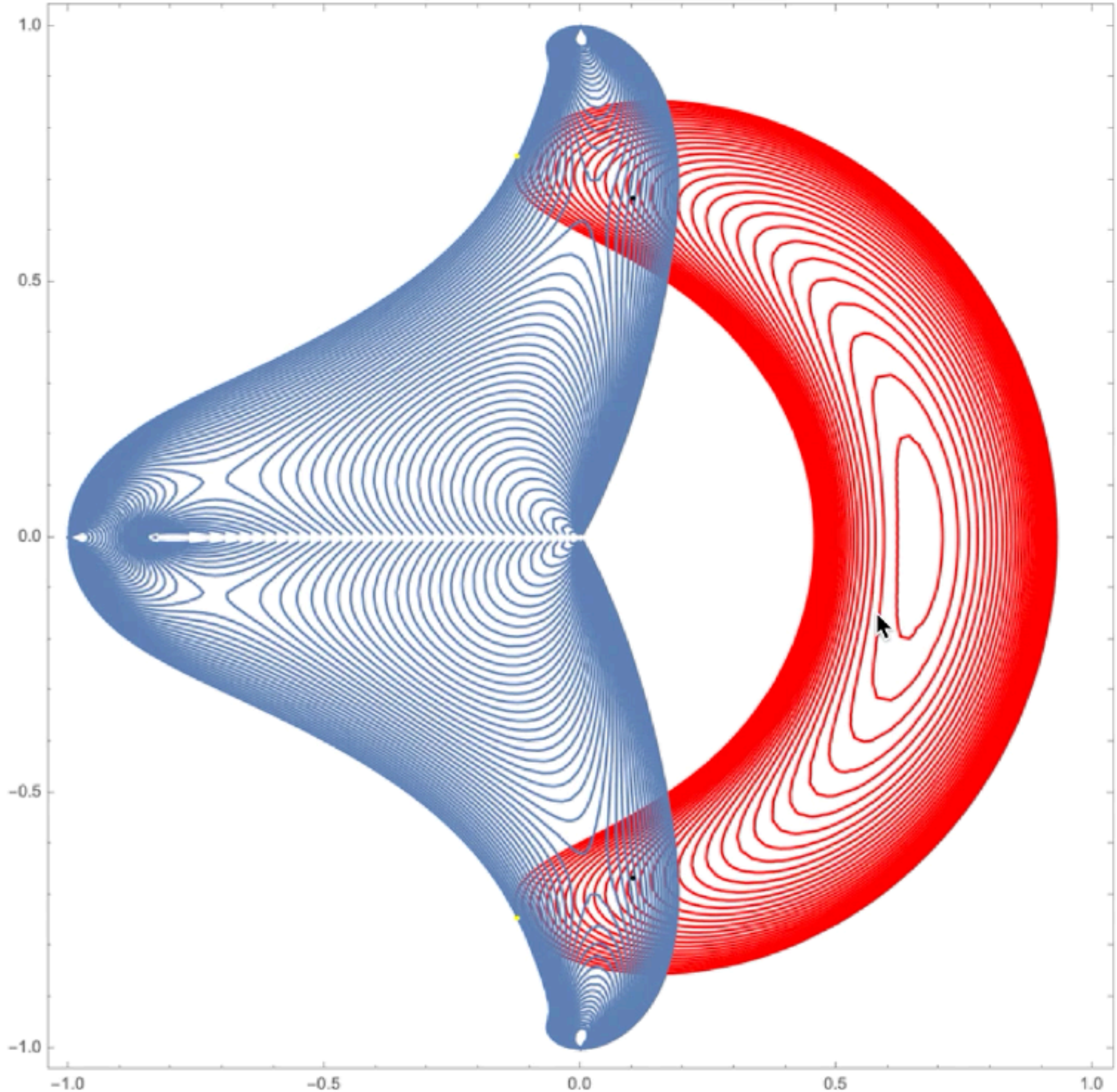


Examples_Binary.nb

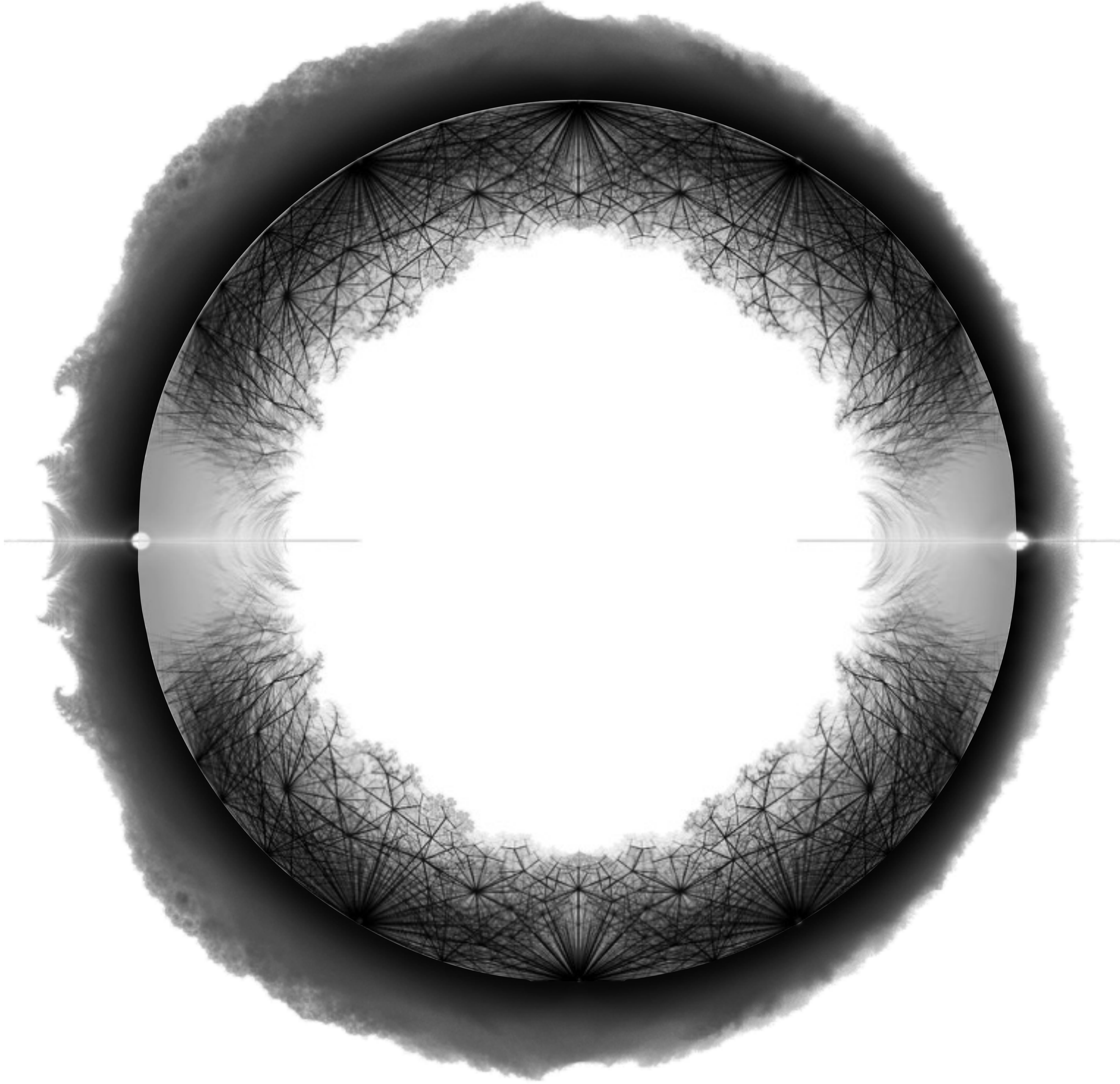
$$\frac{1}{2}\left(-1+\sqrt{-3-4z-4z^2-4z^3}\right)$$ `/. {z -> pt1[[2]] + I pt1[[1]]}`

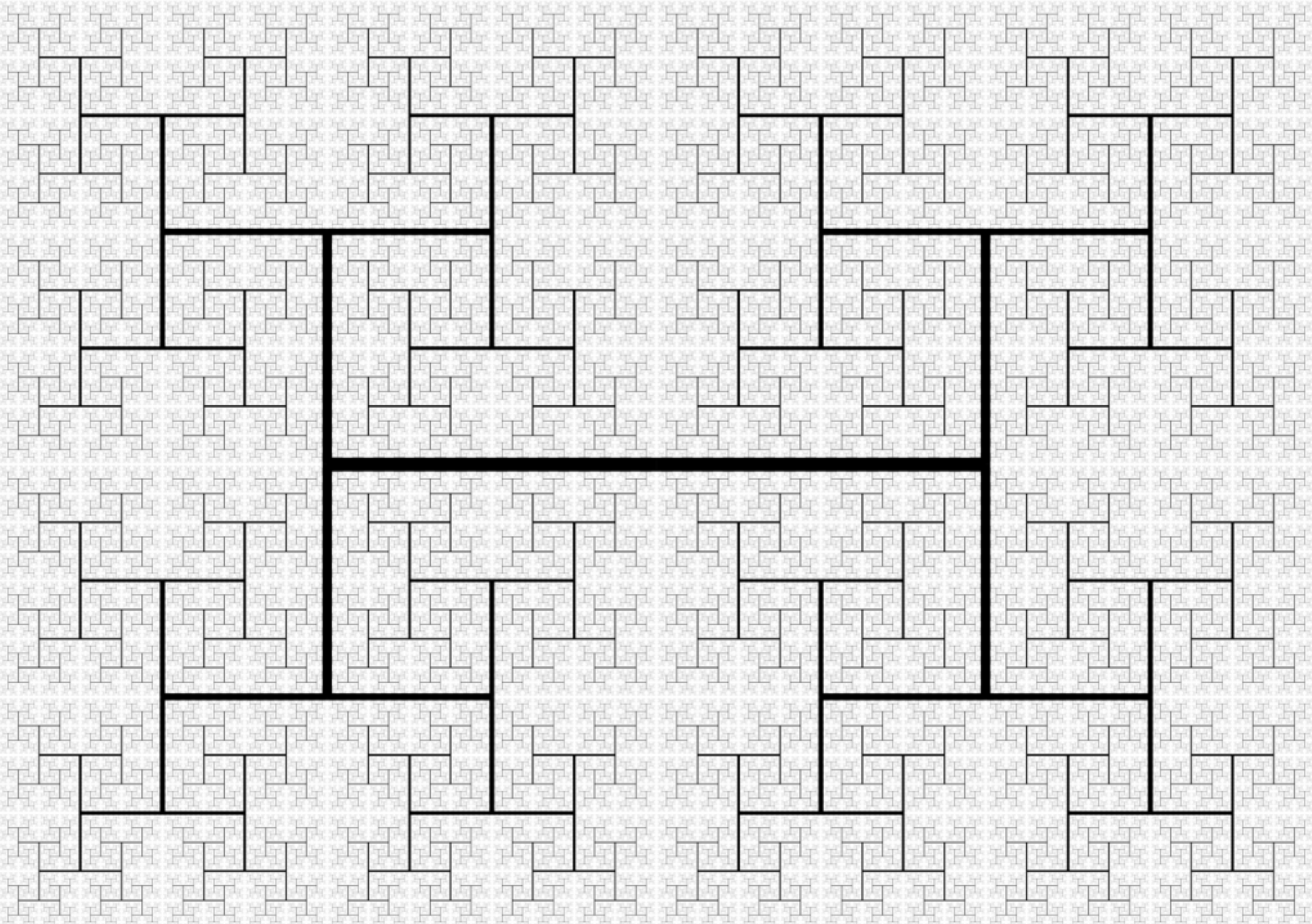
Out[2]=

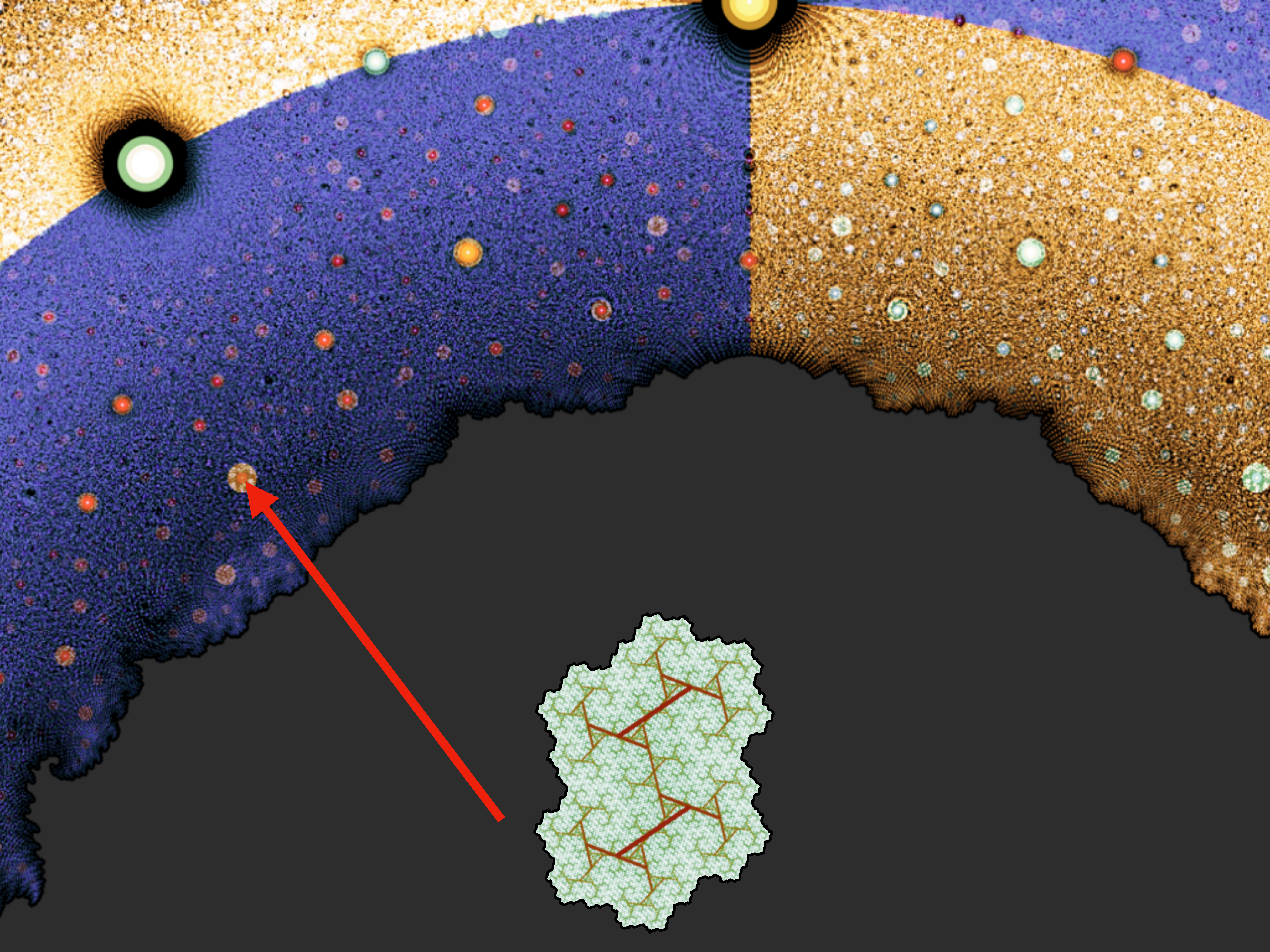


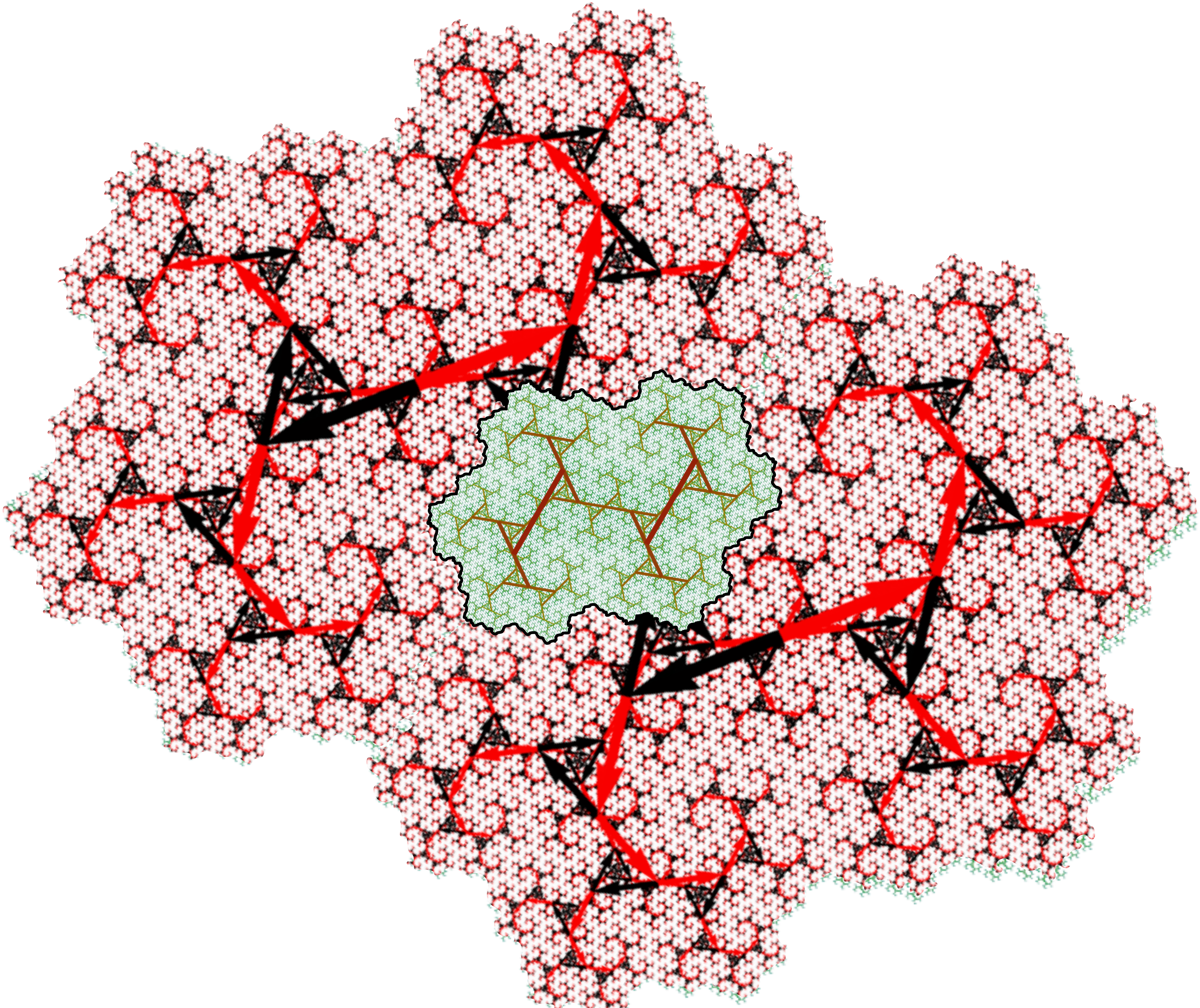


1.0
0.5
0.0
−0.5
−1.0
−1.0
−0.5
0.0
0.5
1.0









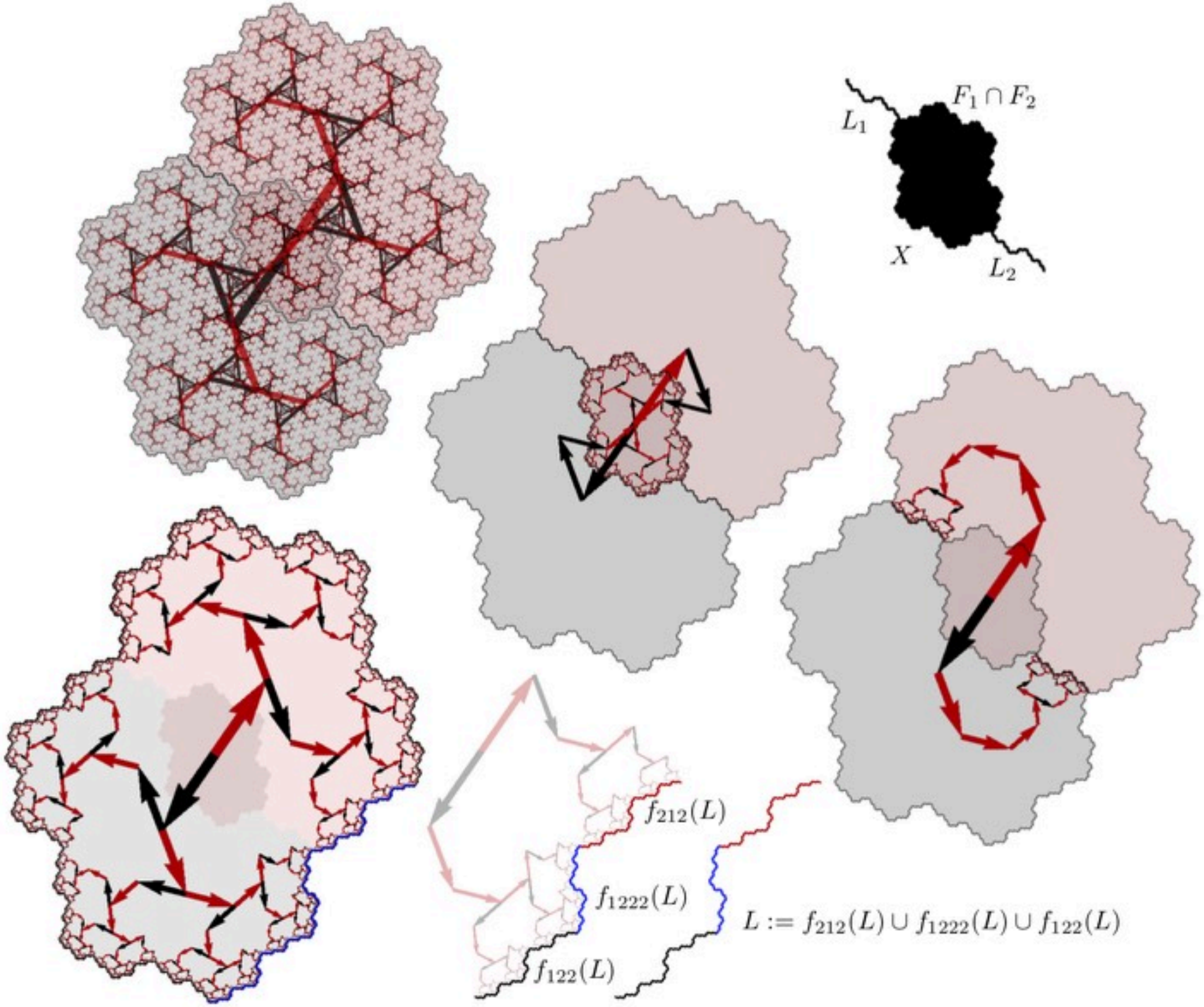
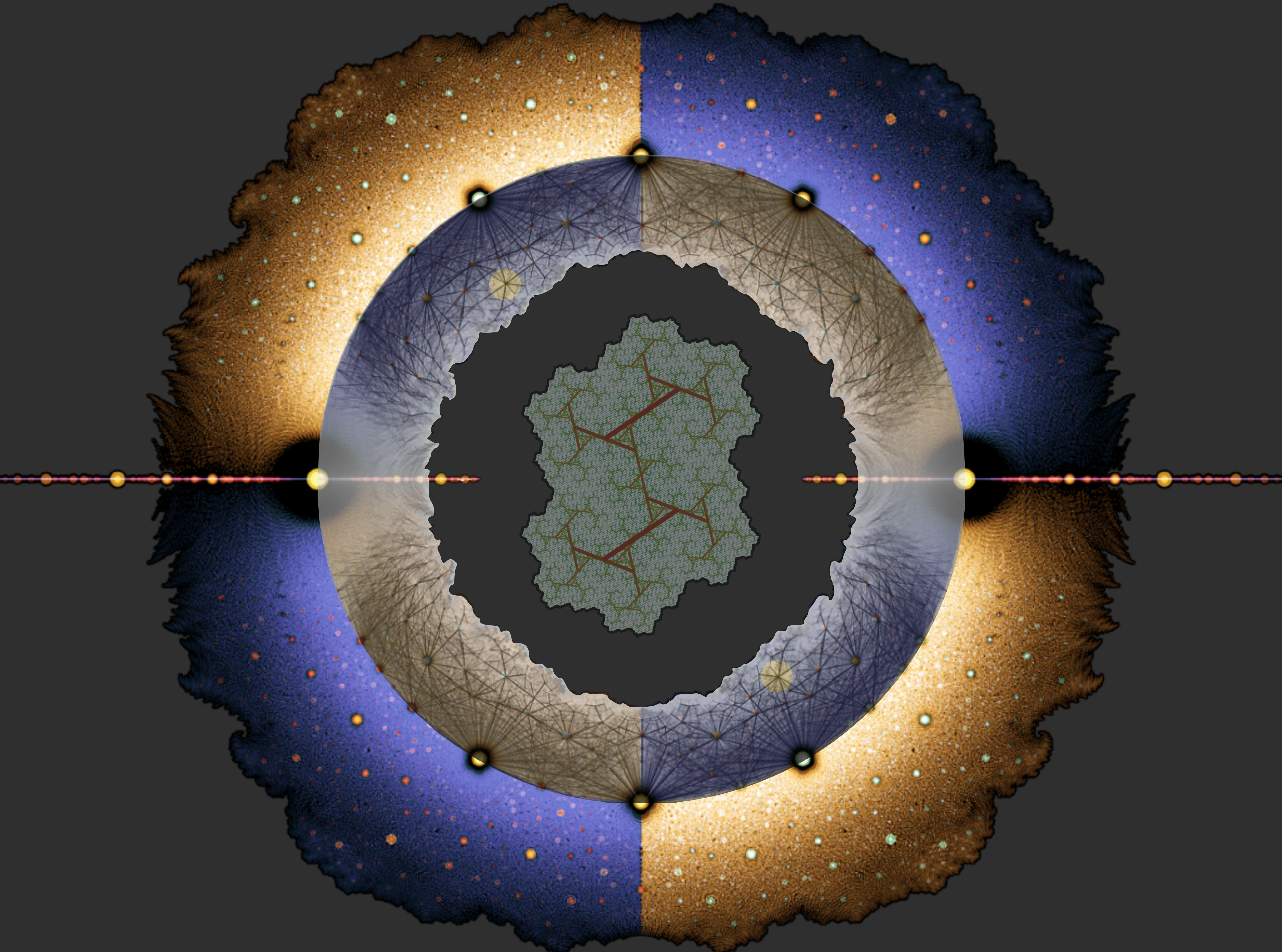
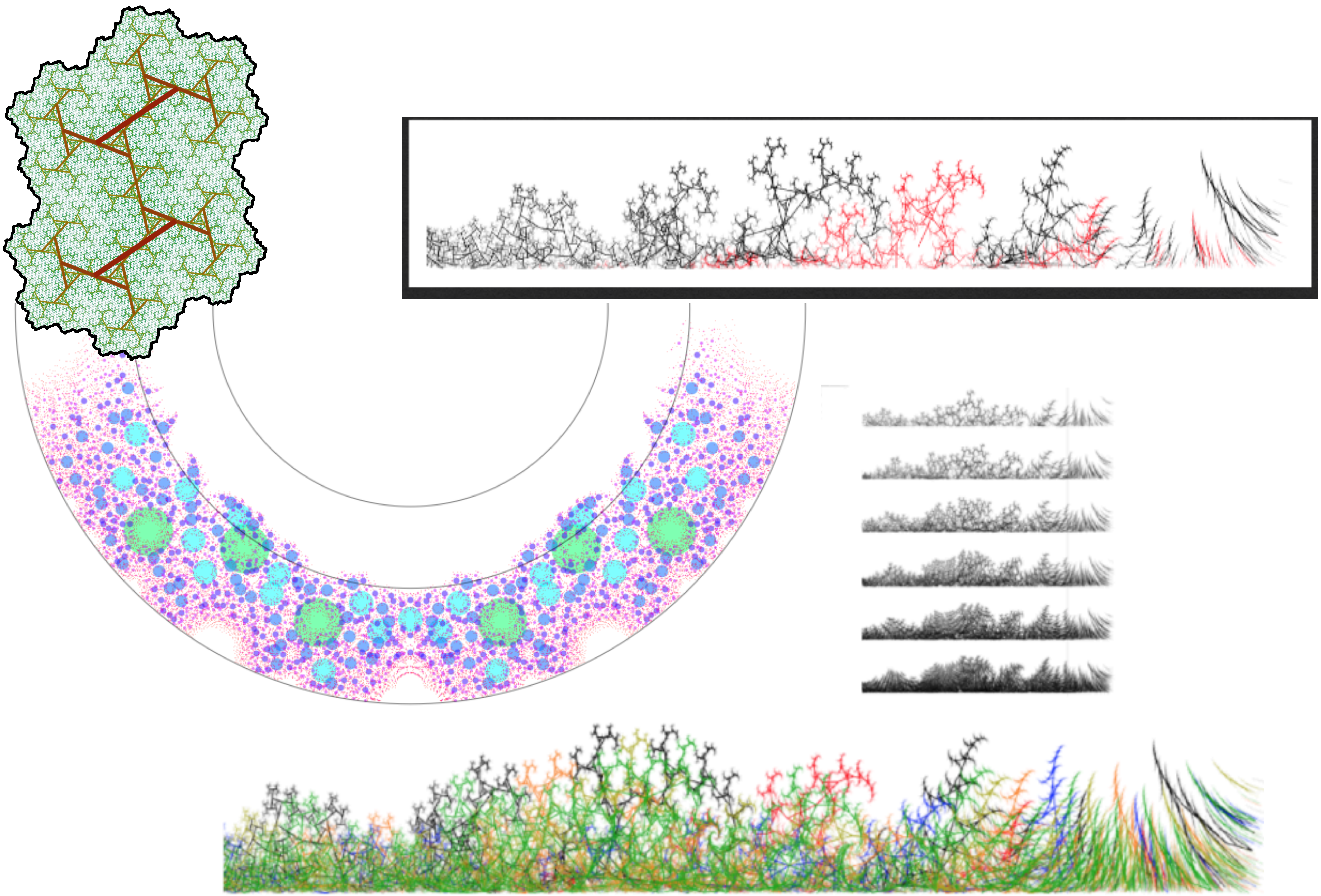


Figure 16: Binary complex tree $T\{c, -c\}$ where $c \approx 0.737e^{-2.176i}$ is a root of $1 + x + x^2 - x^3$. Since we have the following tip-to-tip intersections $\phi(11122) = \phi(21121)$, $\phi(11121) = \phi(21122)$, and $\phi(1112) = \phi(2112)$ theorem 6 tells us that the following piece-to-piece exact overlaps take place $F_{11122} = F_{21121}$, $F_{11121} = F_{21122}$. Moreover $F_{1112} = F_{11121} \cup F_{11122} = F_{21121} \cup F_{21122} = F_{2112}$, i.e. $F_{1112} = F_{2112}$. For this particular example the tipset $F\{c, -c\}$ is the classical Rauzy fractal shown in figure 20 with the particularity of having the first-level pieces F_1 and F_2 intersecting in an non-empty set $F_1 \cap F_2 = L_1 \cup X \cup L_2$ where $X = F_{1112} = F_{2112}$ and L_1 and L_2 are given in terms of the fractal curve $L := f_{212}(L) \cup f_{1222}(L) \cup f_{122}(L)$ as $L_1 = f_{22222}(L)$, $L_2 = f_{12222}(L)$.





Internal structure of *M* illustrated by complex trees' piece-to-piece connectivity roots. 2018