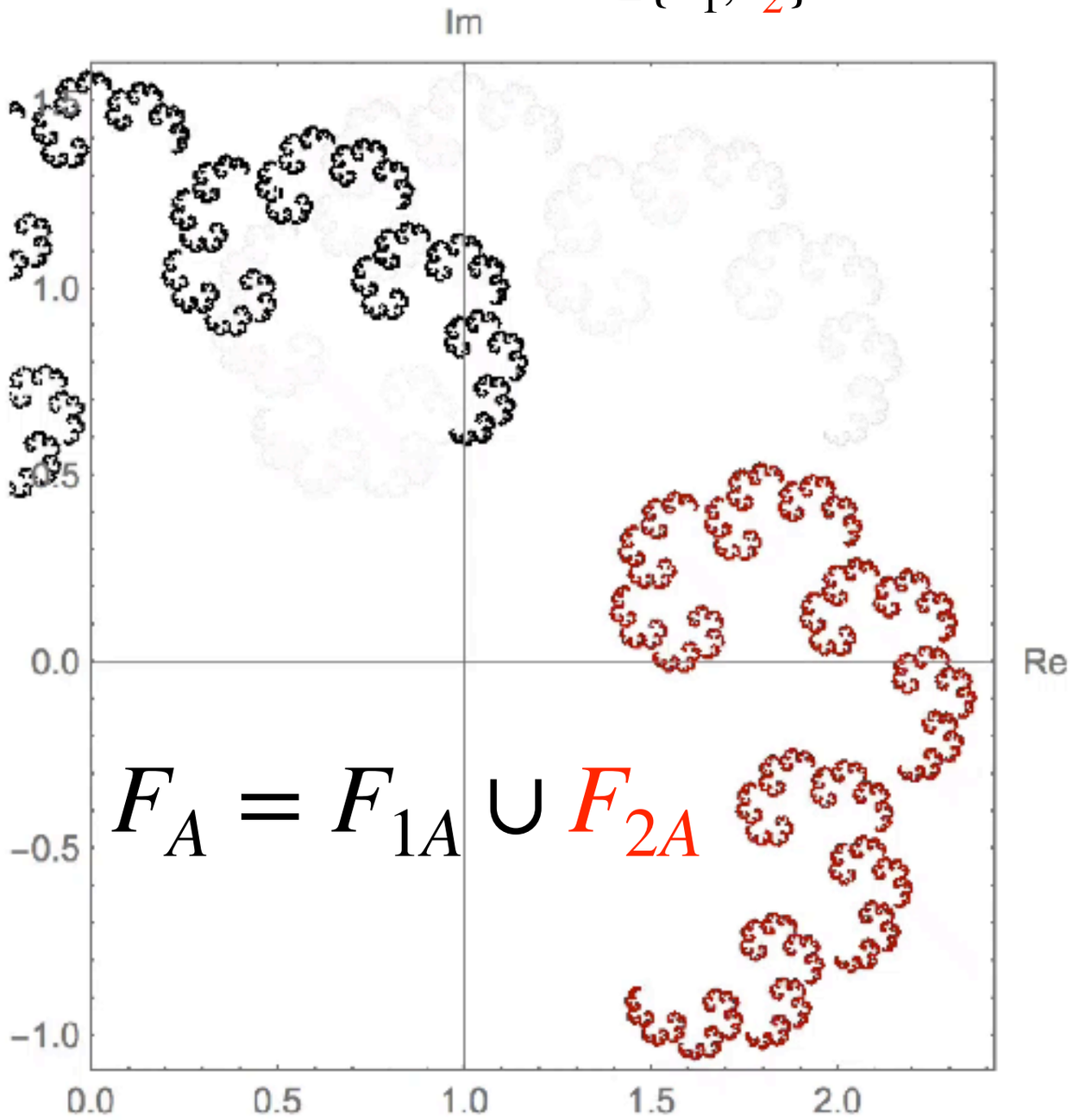
$$F_A = F\{c_1, c_2\} := \bigcup_{w \in \{c_1, c_2\}^\infty} \phi(w)$$



$$F_A = F_{1A} \cup F_{2A}$$

The tipset is a self-similar set

$$\{c_1, c_2\} = \{.3 + i.6,\ .5 - i.3\}$$

$$F_A = f_1(F_A) \cup f_2(F_A)$$

First-level pieces

$$F_{1A} := f_1(F_A) = 1 + c_1 \cdot F_A$$

$$F_{2A} := f_2(F_A) = 1 + c_2 \cdot F_A$$

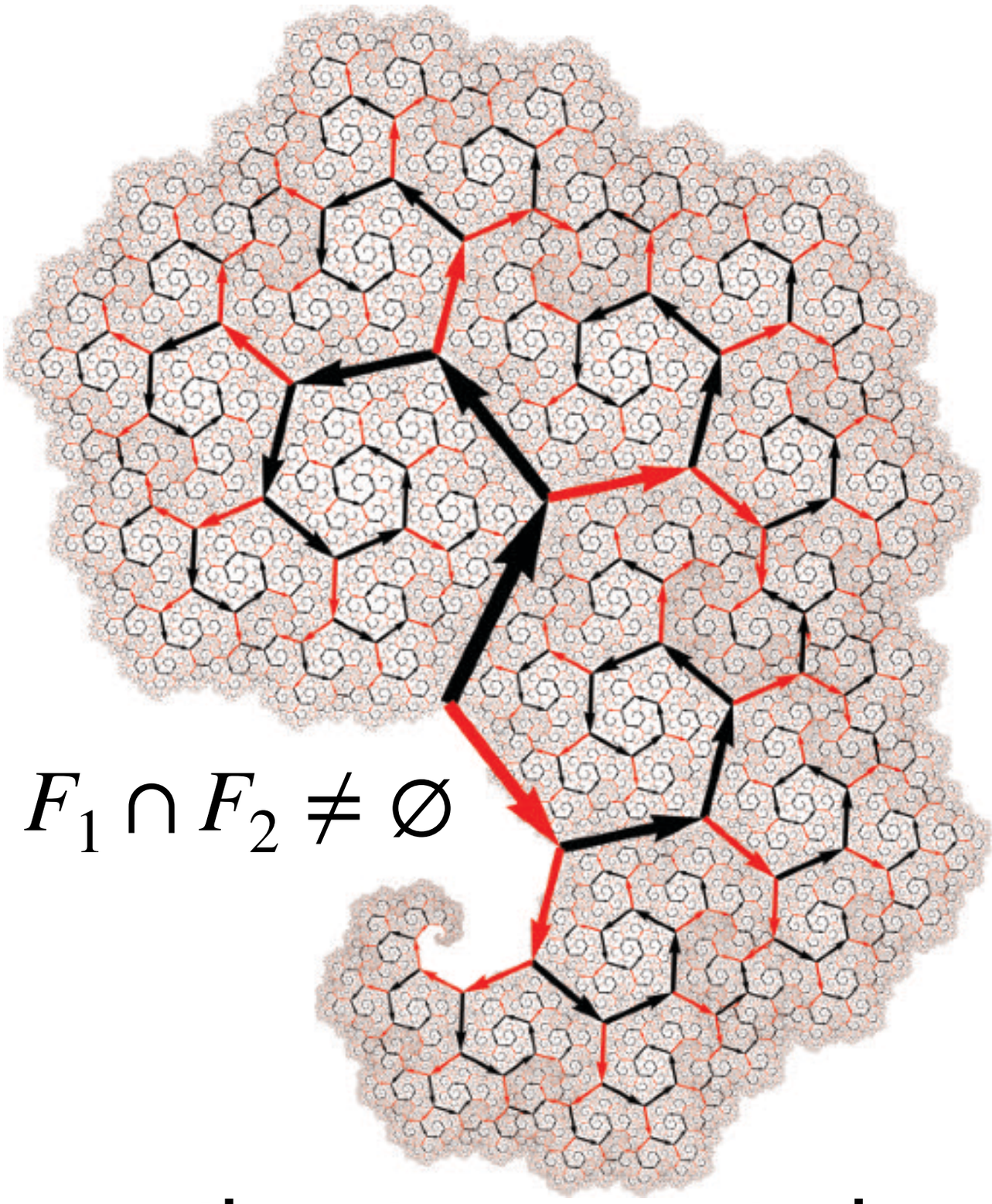
$$r_2 = |c_2| \approx 0.58$$

$$\theta_2 = \arg(c_2) \approx -31^\circ$$

$$r_1 = |c_1| \approx 0.67$$

$$\theta_1 = \arg(c_1) \approx 63^\circ$$

$$F_{1A} \cap F_{2A} = \varnothing$$

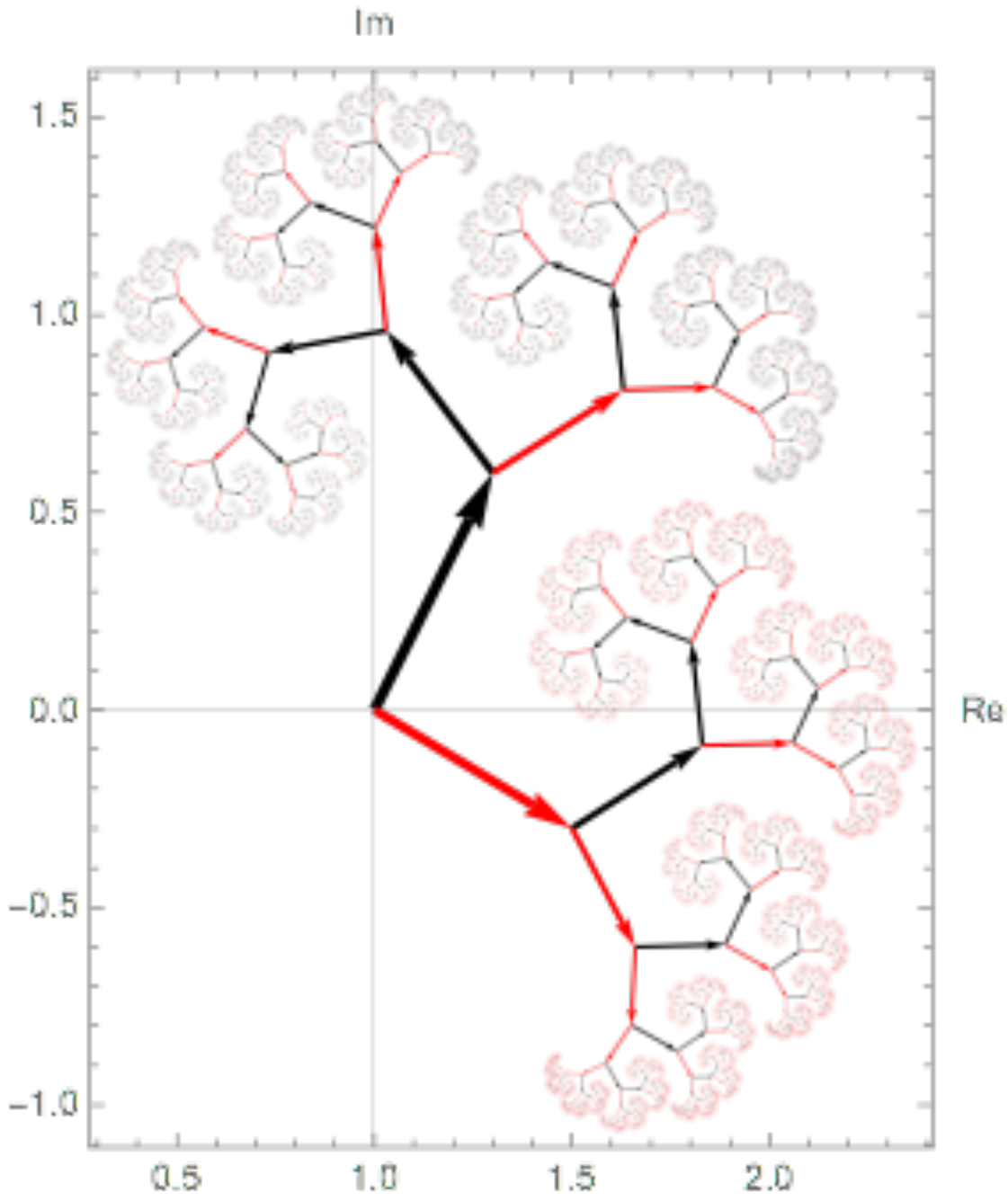


$$F_1 \cap F_2 \neq \emptyset$$

tipset connected

$$T\{0.352 + i0.720, 0.395 - i0.507\}$$

$$F_1 \cap F_2 = \emptyset$$



tipset disconnected

$$T\{0.3 + i0.6, 0.5 - i0.3\}$$

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