Illustrating Mathematics, ICERM, Brown University, October 2019

Unfolding Complex Trees with n-fold rotational symmetry

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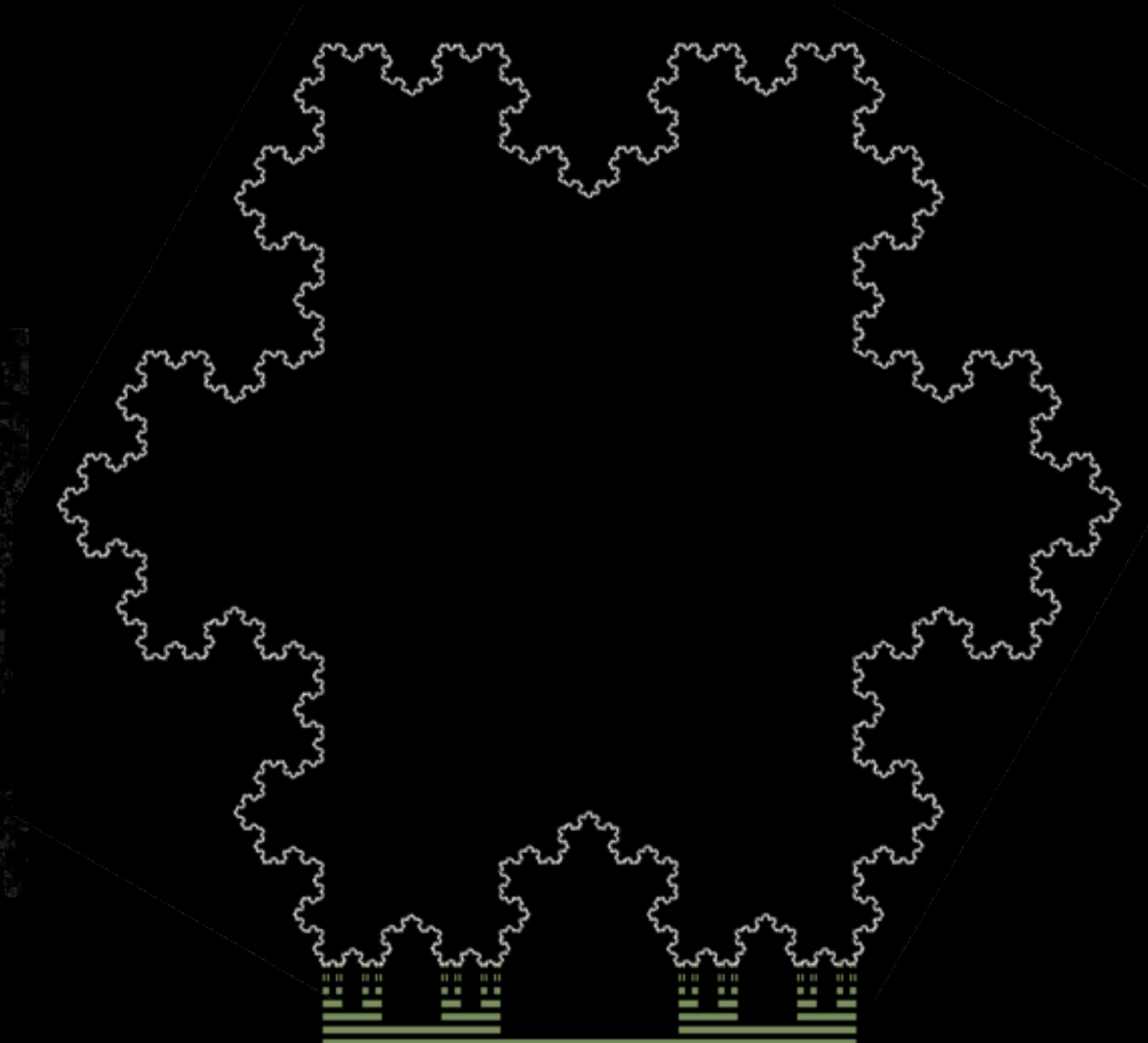
www.ComplexTrees.com

Xavier Jarque & Núria Fagella

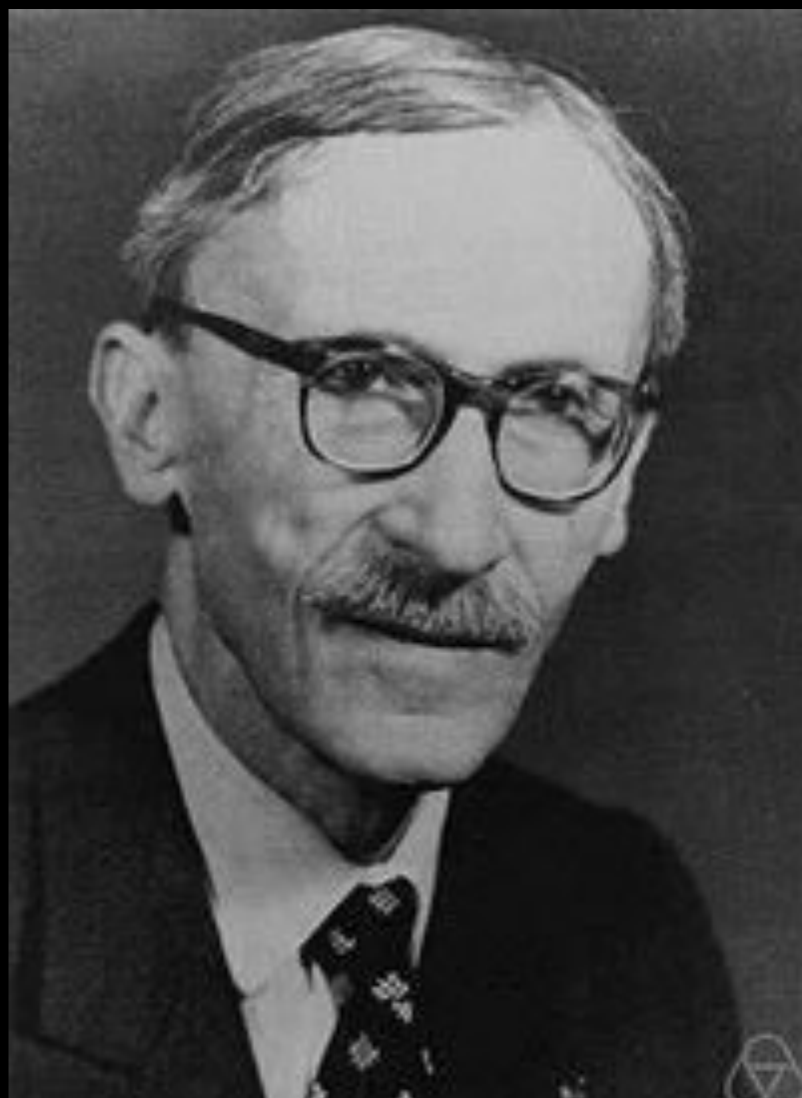
Helge von Koch
(1870 –1924)

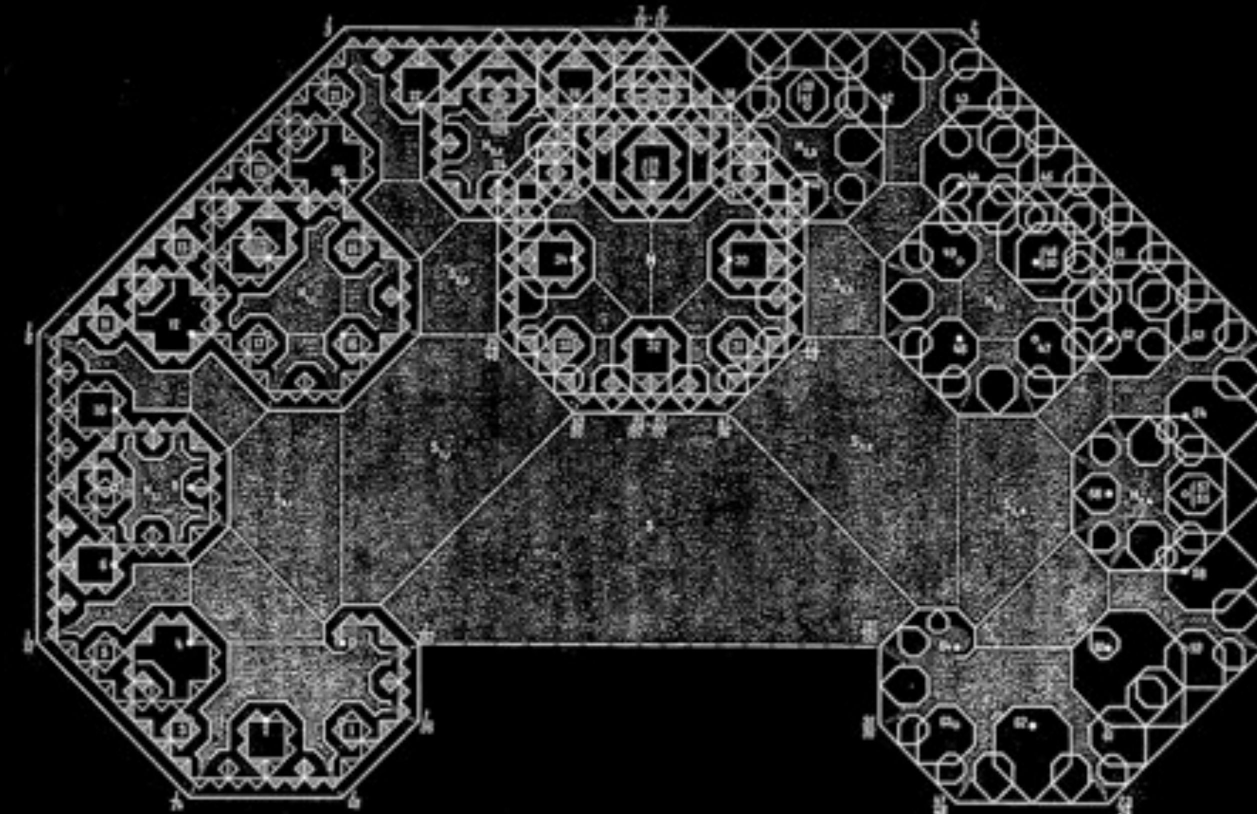


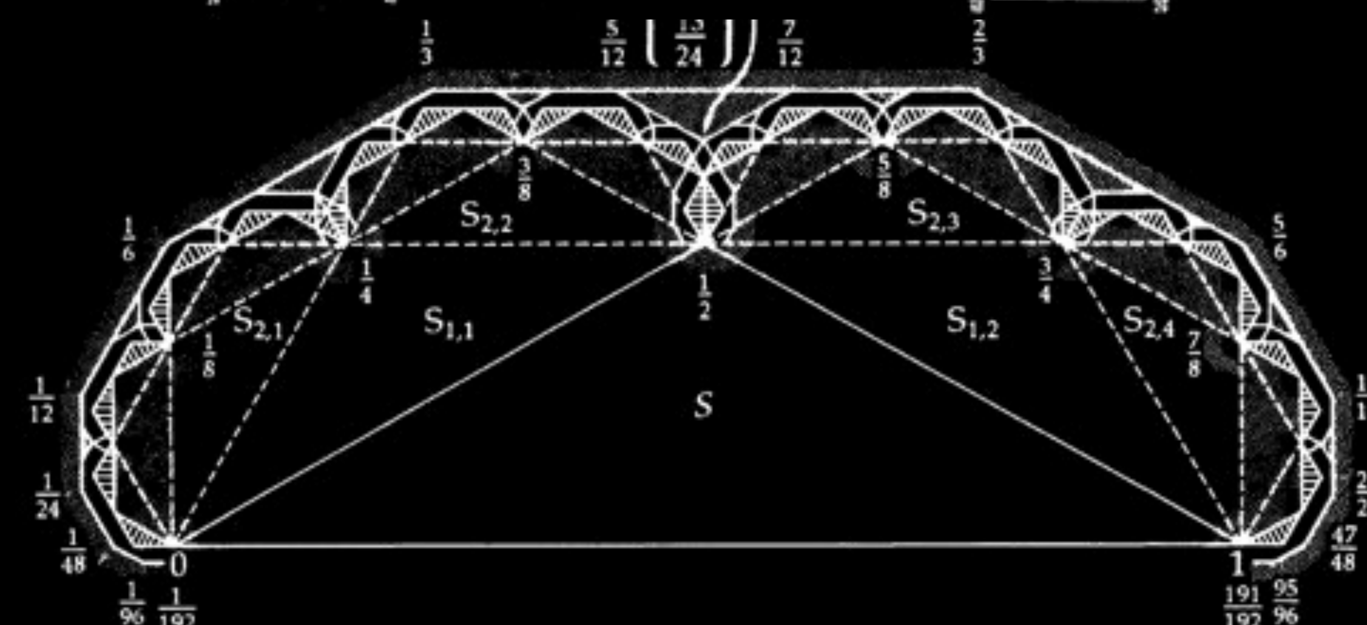
"Sur une courbe continue sans tangente, obtenue par une construction géométrique élémentaire."

Archiv för Matemat., Astron. och Fys. 1, 681-702, 1904.

Paul Lévy
(1886 –1971)

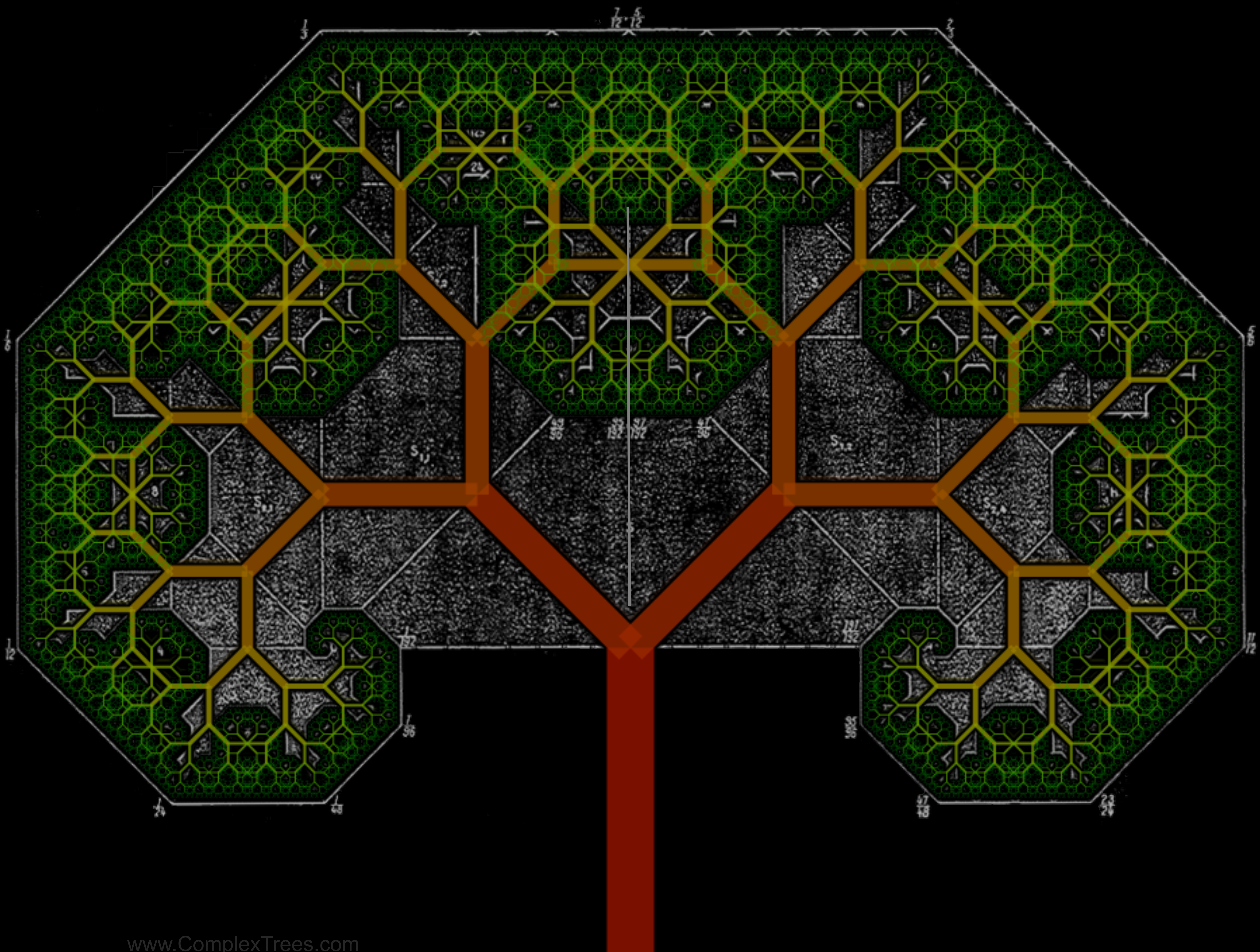


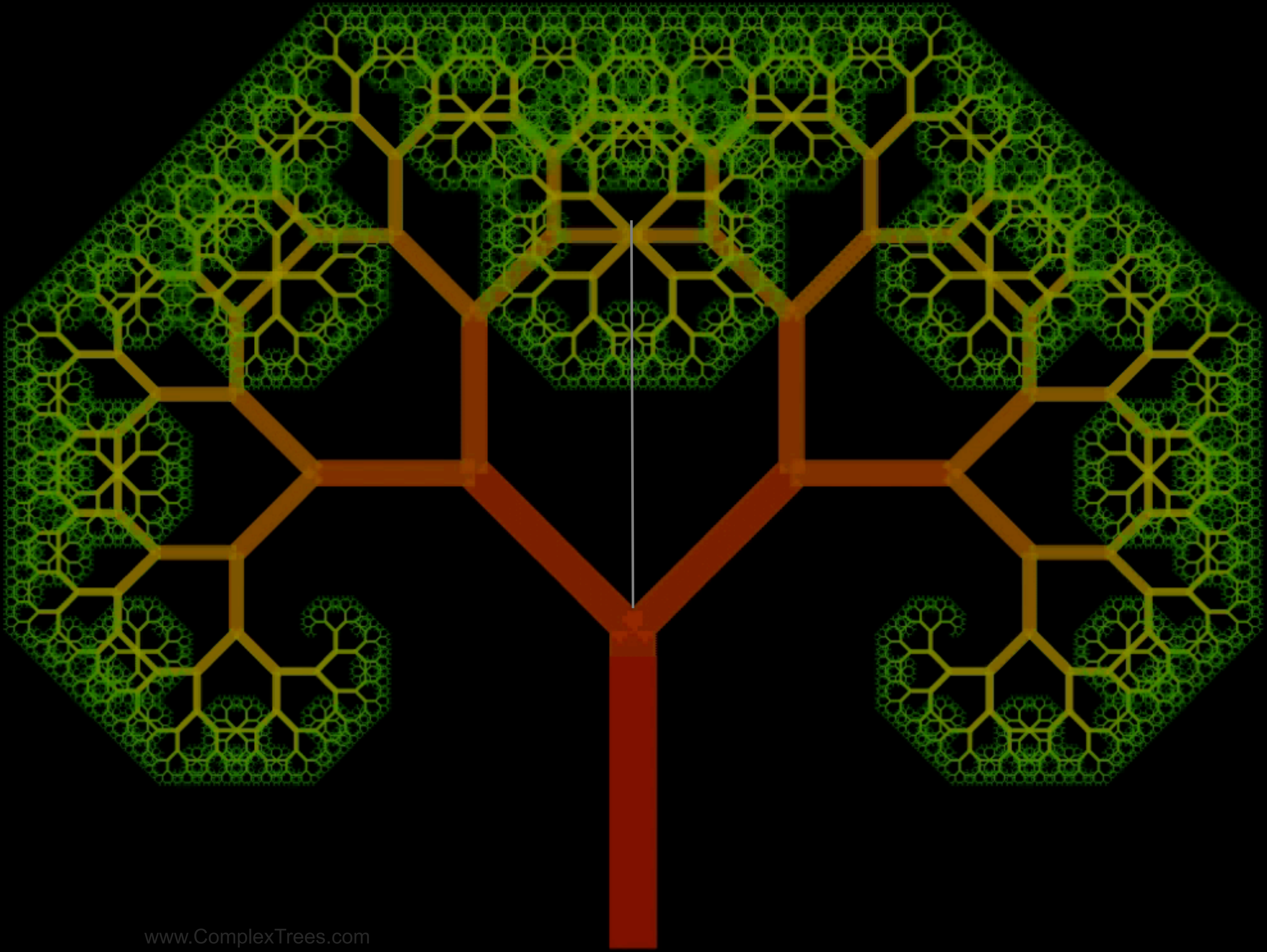


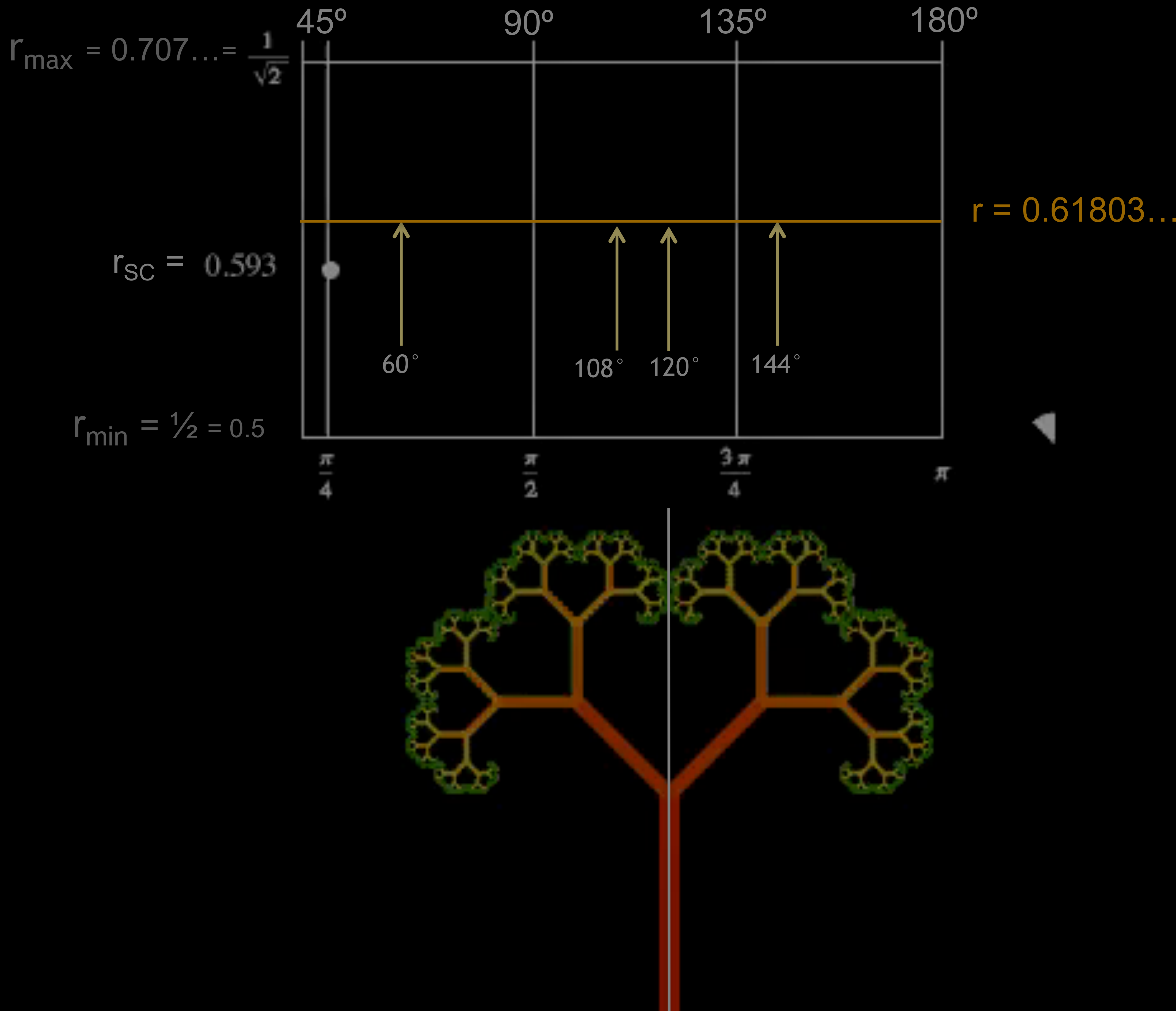


"Les courbes planes ou gauches et les surfaces composées de parties semblales au tout. »

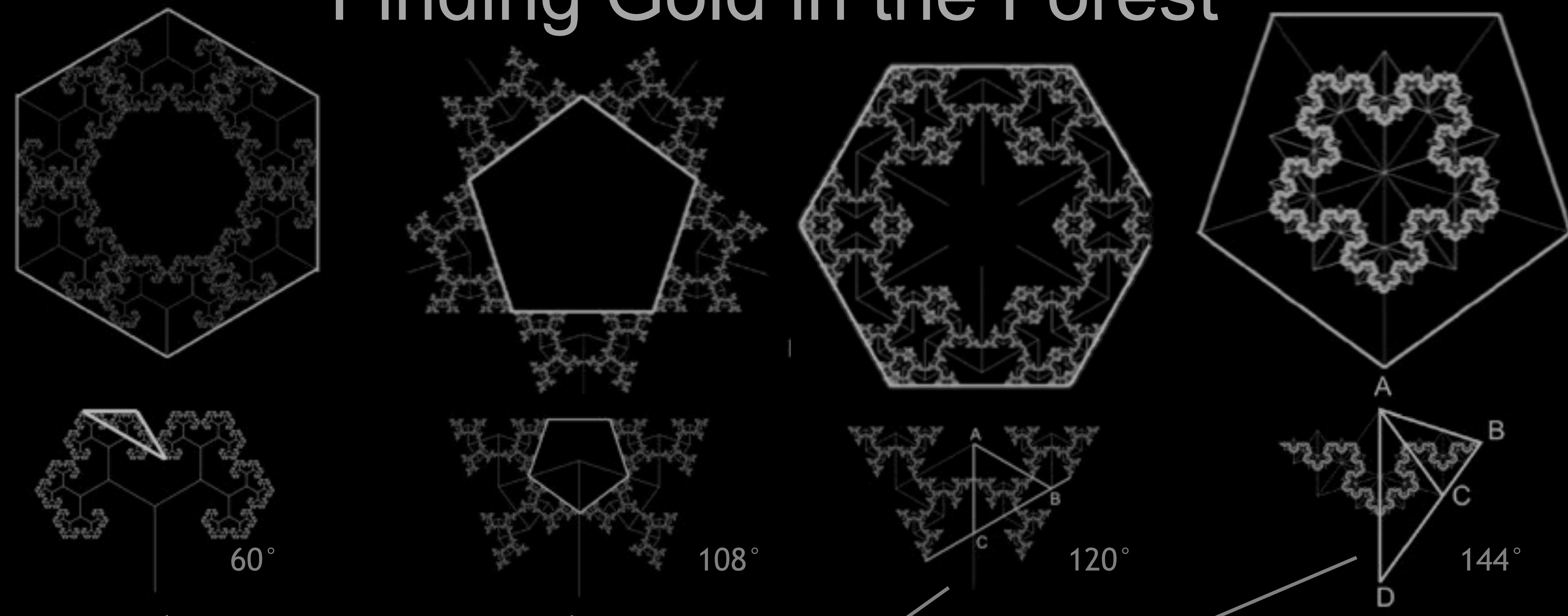
J. l'École Polytech., 227-247 and 249-291, 1939.

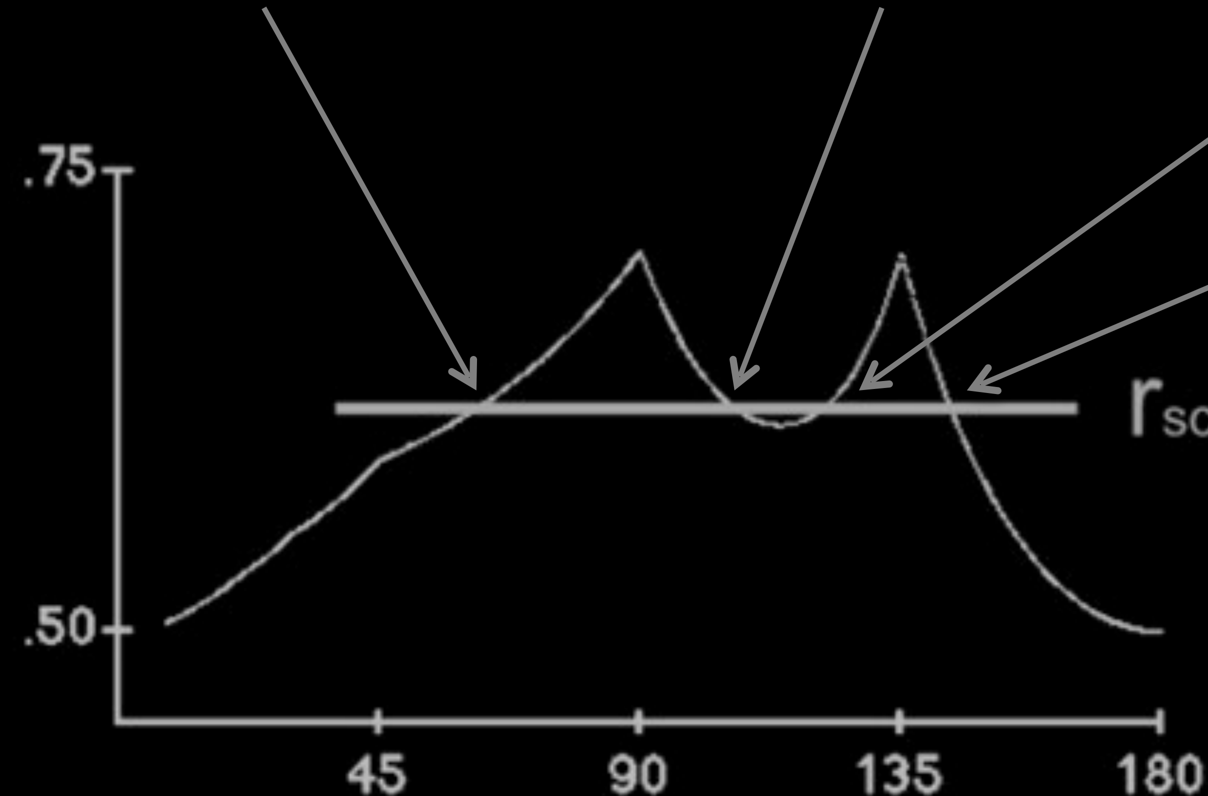





45º
90º
135º
180º
$r_{max} = 0.707\ldots = \frac{1}{\sqrt{2}}$
$r = 0.61803\ldots$
$r_{SC} = 0.593$
60°
108°
120°
144°
$r_{min} = ½ = 0.5$
$\frac{\pi}{4}$
$\frac{\pi}{2}$
$\frac{3\pi}{4}$
π

“Finding Gold in the Forest”





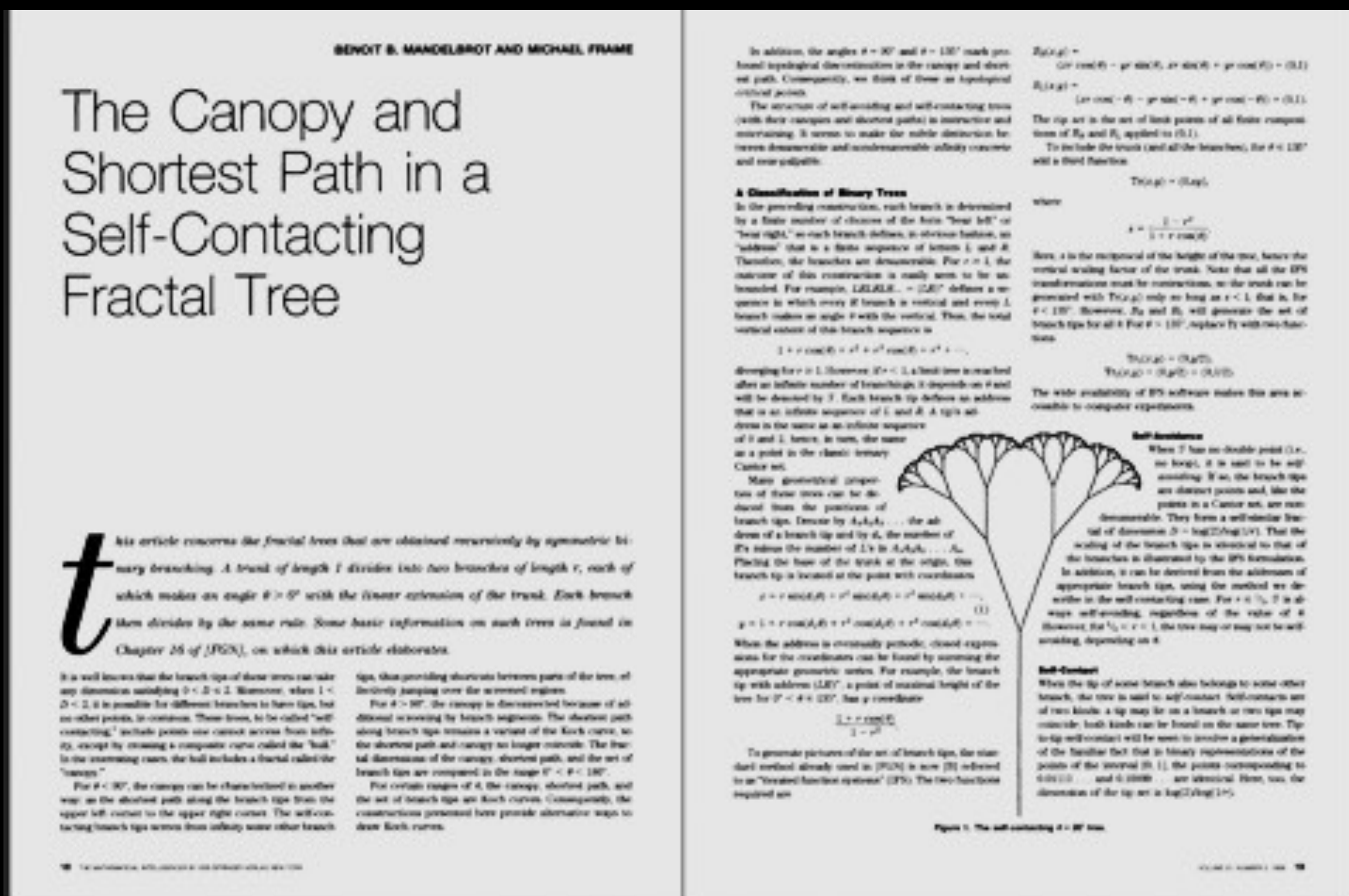
θ

$r_{sc} = 1/\Phi = \Phi - 1 = (1+\sqrt{5})½ - 1 \approx 0.618…$

The Beauty of Fractals: Six Different Views
By Dennis Gulick, Jon Scott MAA, Feb 28, 2011
“Excursions Through a Forest of Golden Fractal Trees”
Tara D. Taylor, St. Francis Xavier University

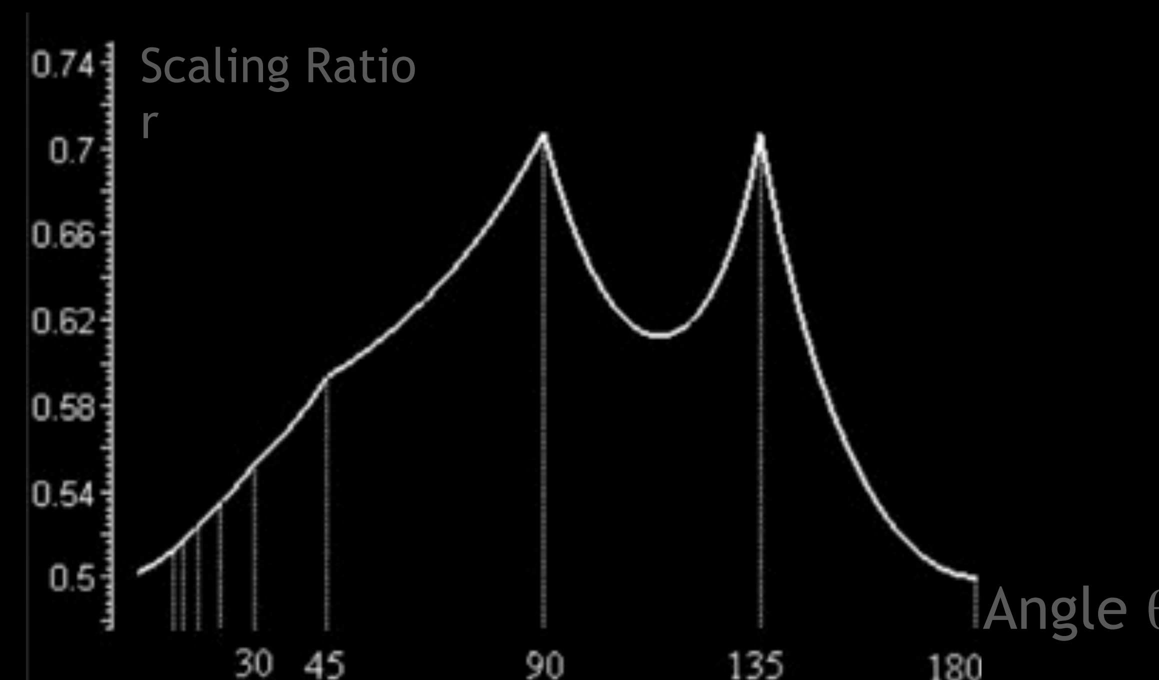
Self-Contacting Binary Fractal Trees

Michael Frame and Benôit B. Mandelbrot
The Mathematical Intelligencer, 1999 Springer-Verlag


BENOIT B. MANDELBROT AND MICHAEL FRAME

The Canopy and Shortest Path in a Self-Contacting Fractal Tree

Mathematical Details for the article of Frame & Mandelbrot
Self-Contacting Fractal Trees
by Prof. Donald C. West © (1999)



http://faculty.plattsburgh.edu/don.west/trees/index.htm

Tara D. Taylor
"Computational Topology and Fractal Trees",
PhD Thesis, Dalhousie University, Canada (2005)

The Beauty of Fractals: Six Different Views
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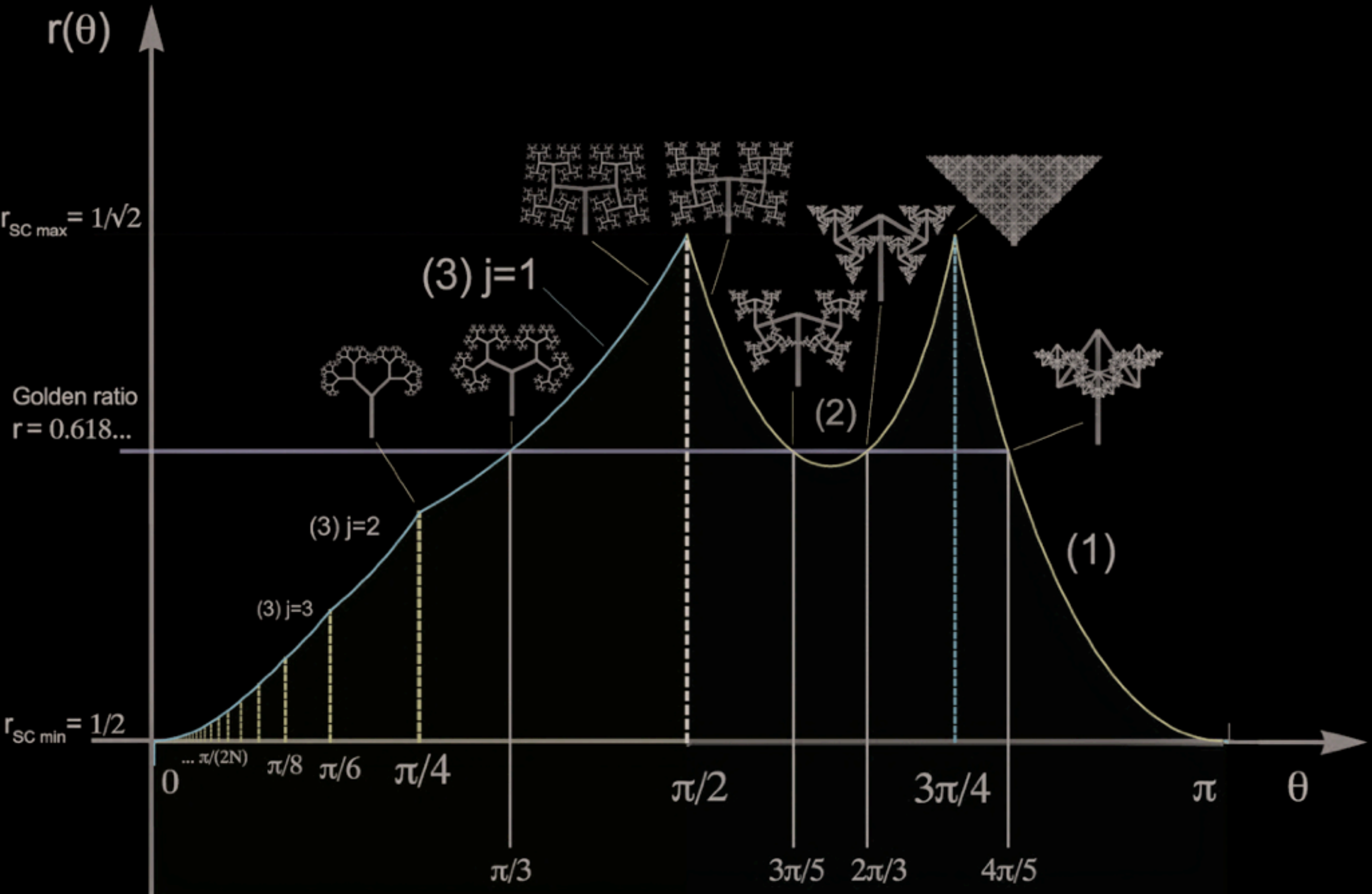
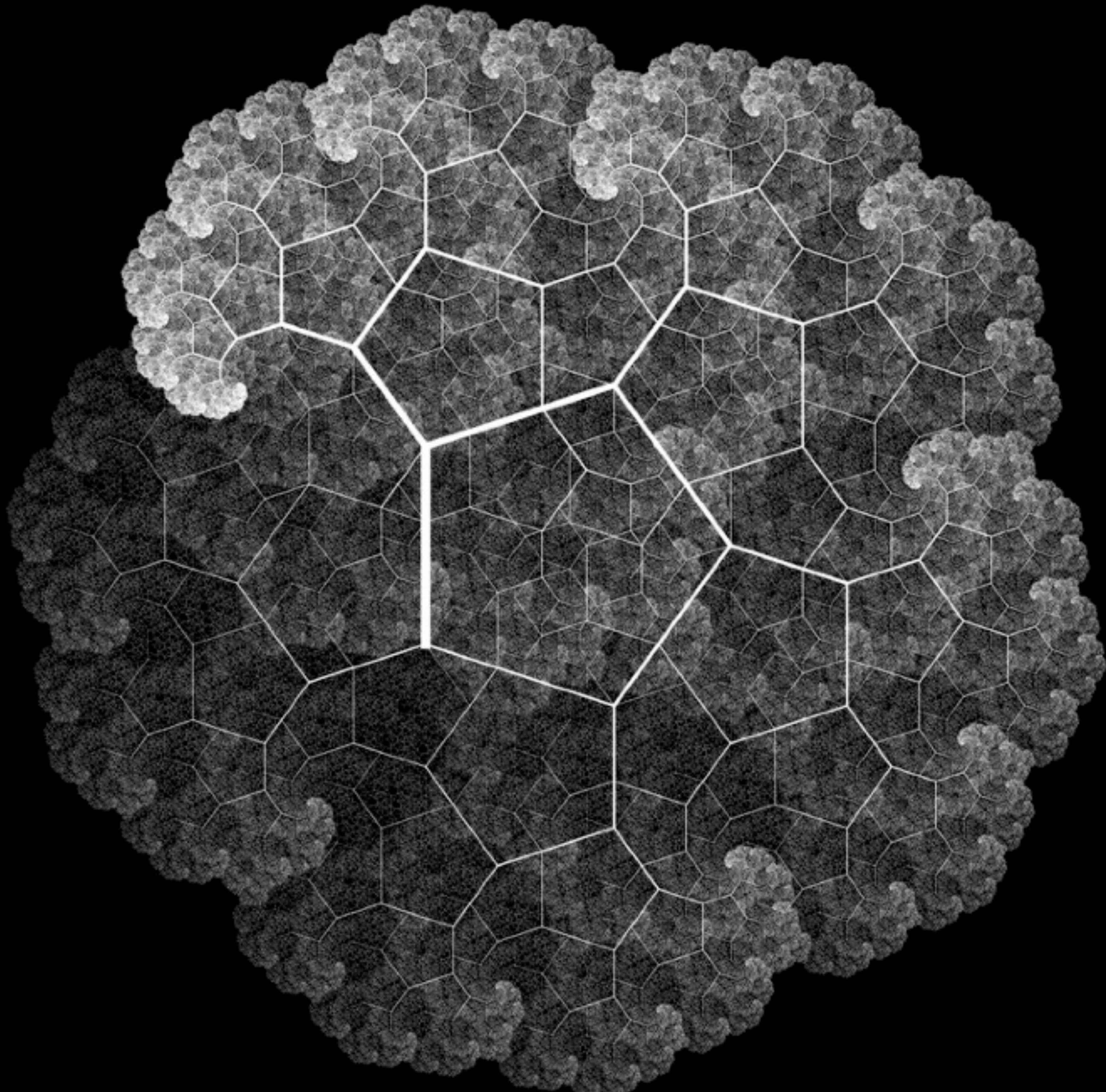
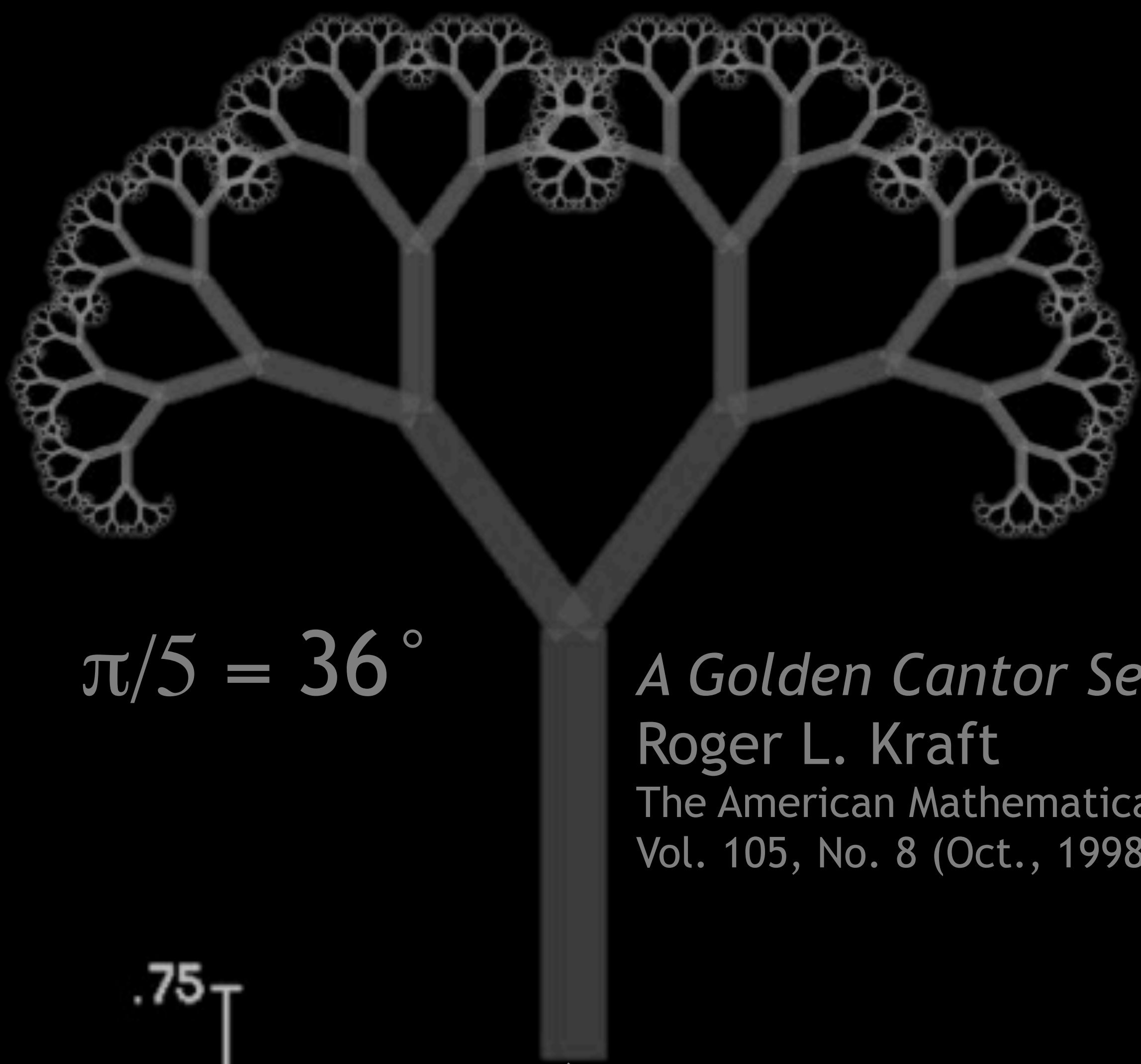


r(θ)
$r_{SC\ max} = 1/\sqrt{2}$
Golden ratio
r = 0.618...
$r_{SC\ min} = 1/2$
(3) j=1
(3) j=2
(3) j=3
(2)
(1)
0
... π/(2N)
π/8
π/6
π/4
π/2
3π/4
π
θ
π/3
3π/5
2π/3
4π/5

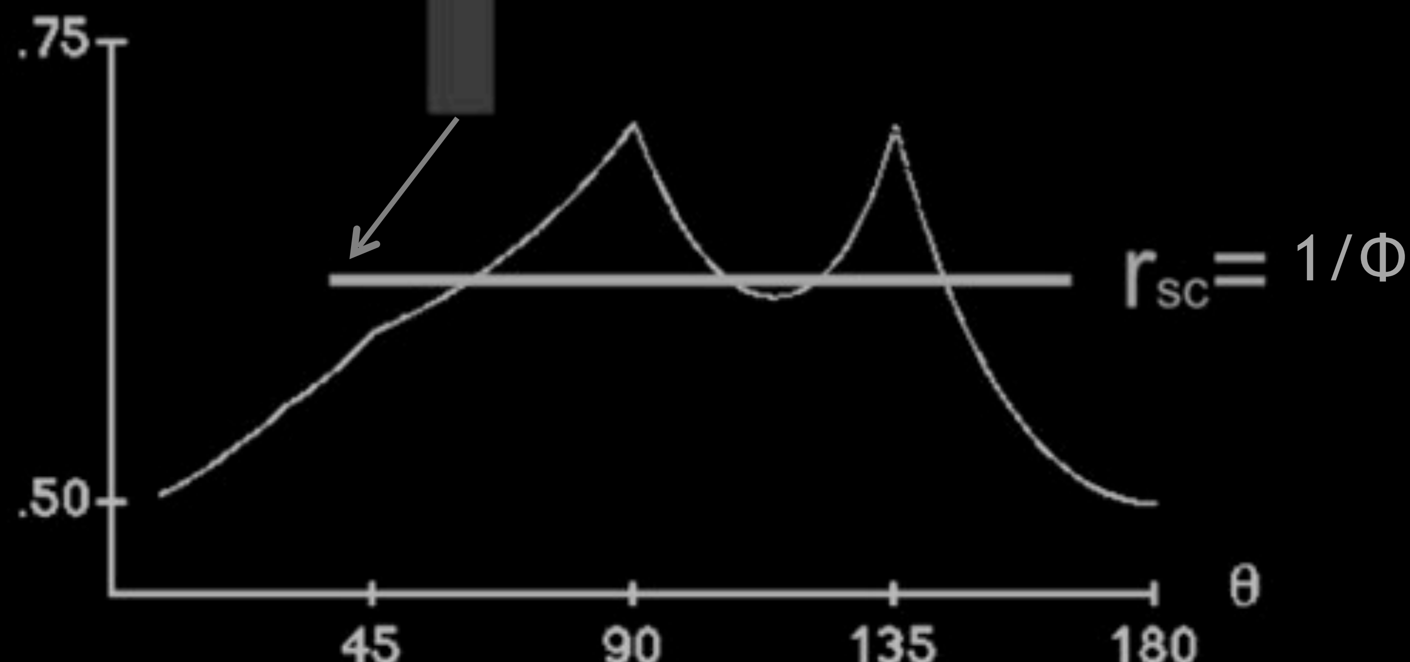


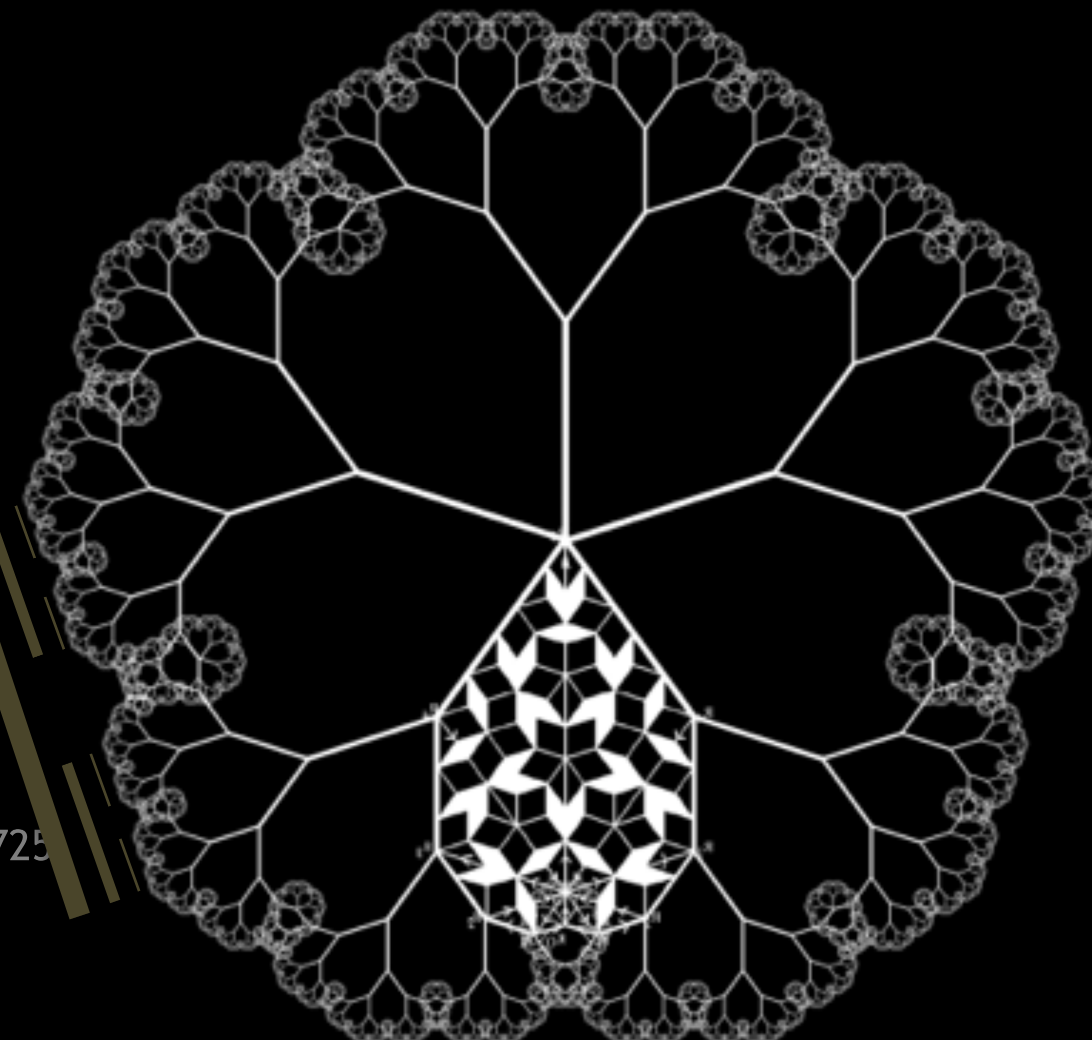
A Hidden Golden Tree



$\pi/5 = 36^\circ$

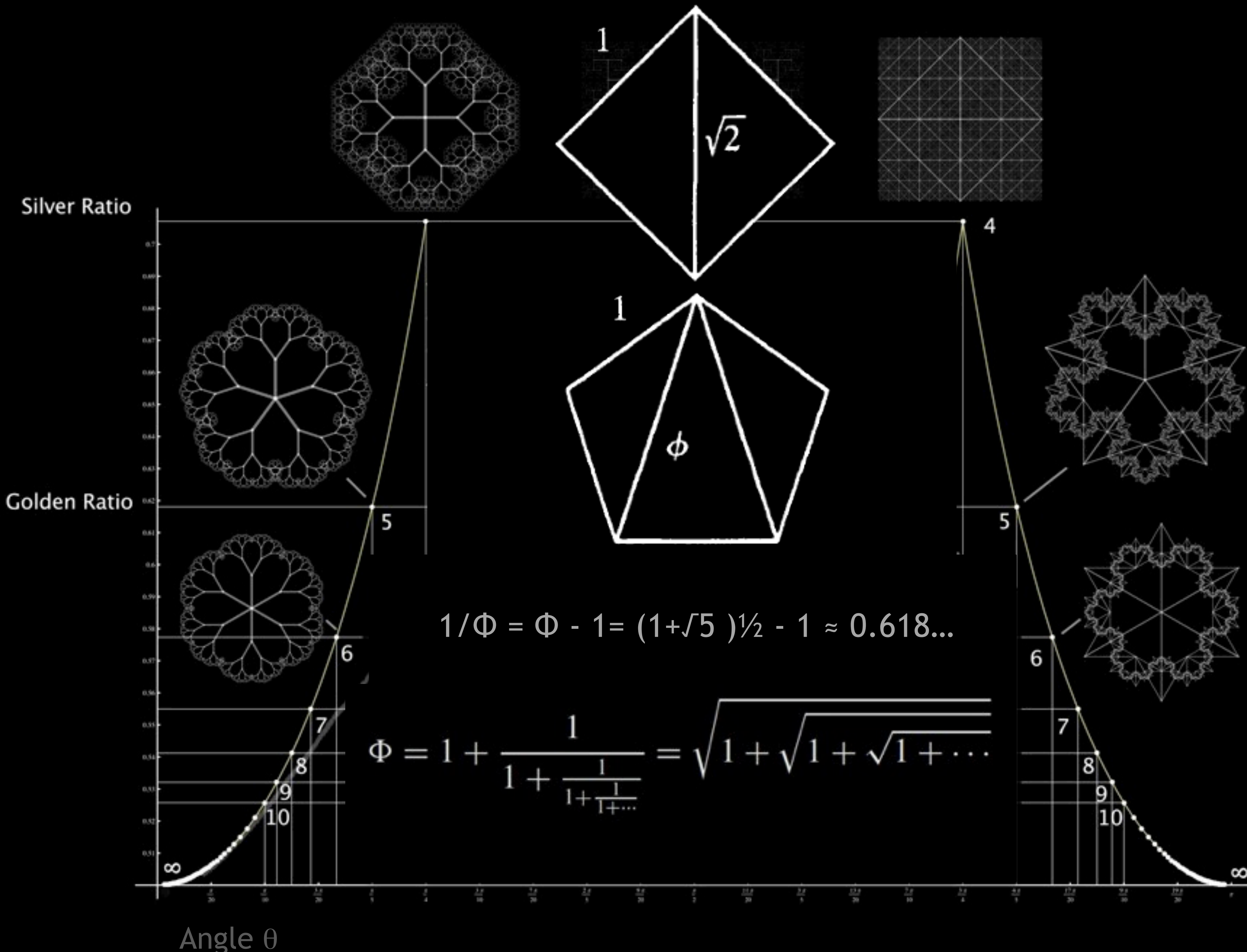
A Golden Cantor Set
Roger L. Kraft
The American Mathematical Monthly,
Vol. 105, No. 8 (Oct., 1998), pp. 718-725





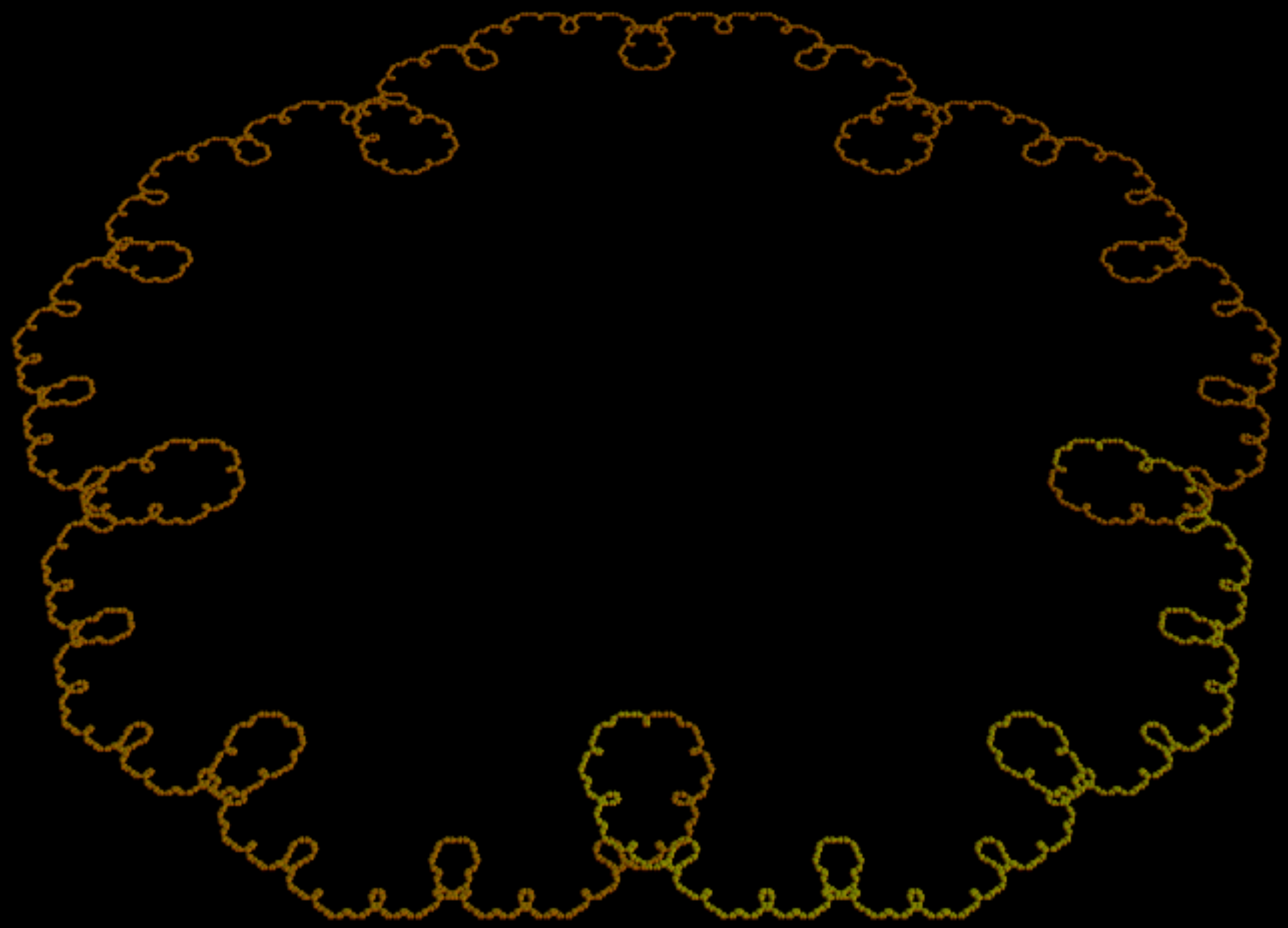
Nonlinear Dynamics and Chaos: Applications in Atmospheric Sciences
A.M.Selvam

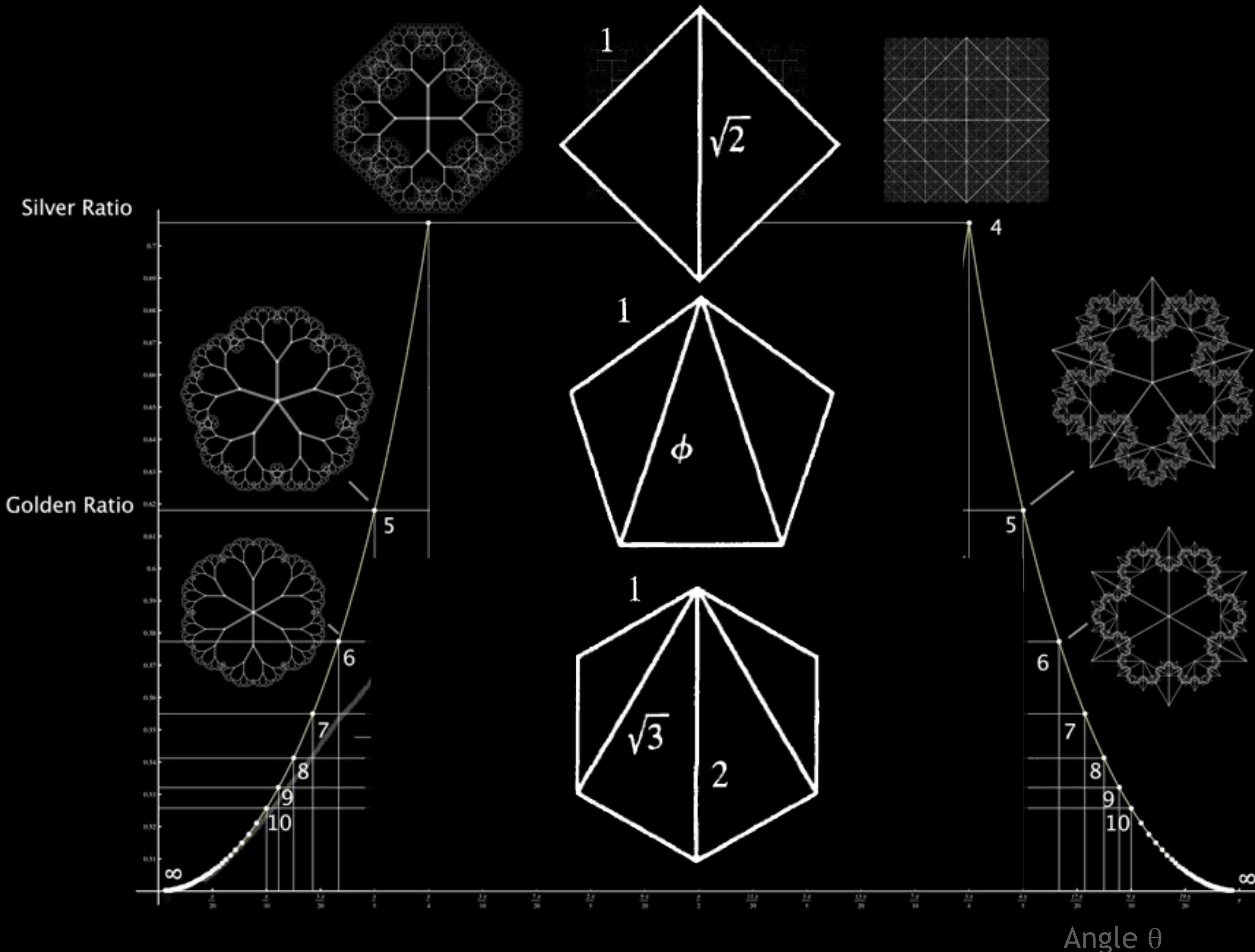
Fig. 6. Quasicrystalline structure of the quasiperiodic Penrose tiling pattern and Fibonacci sequence.



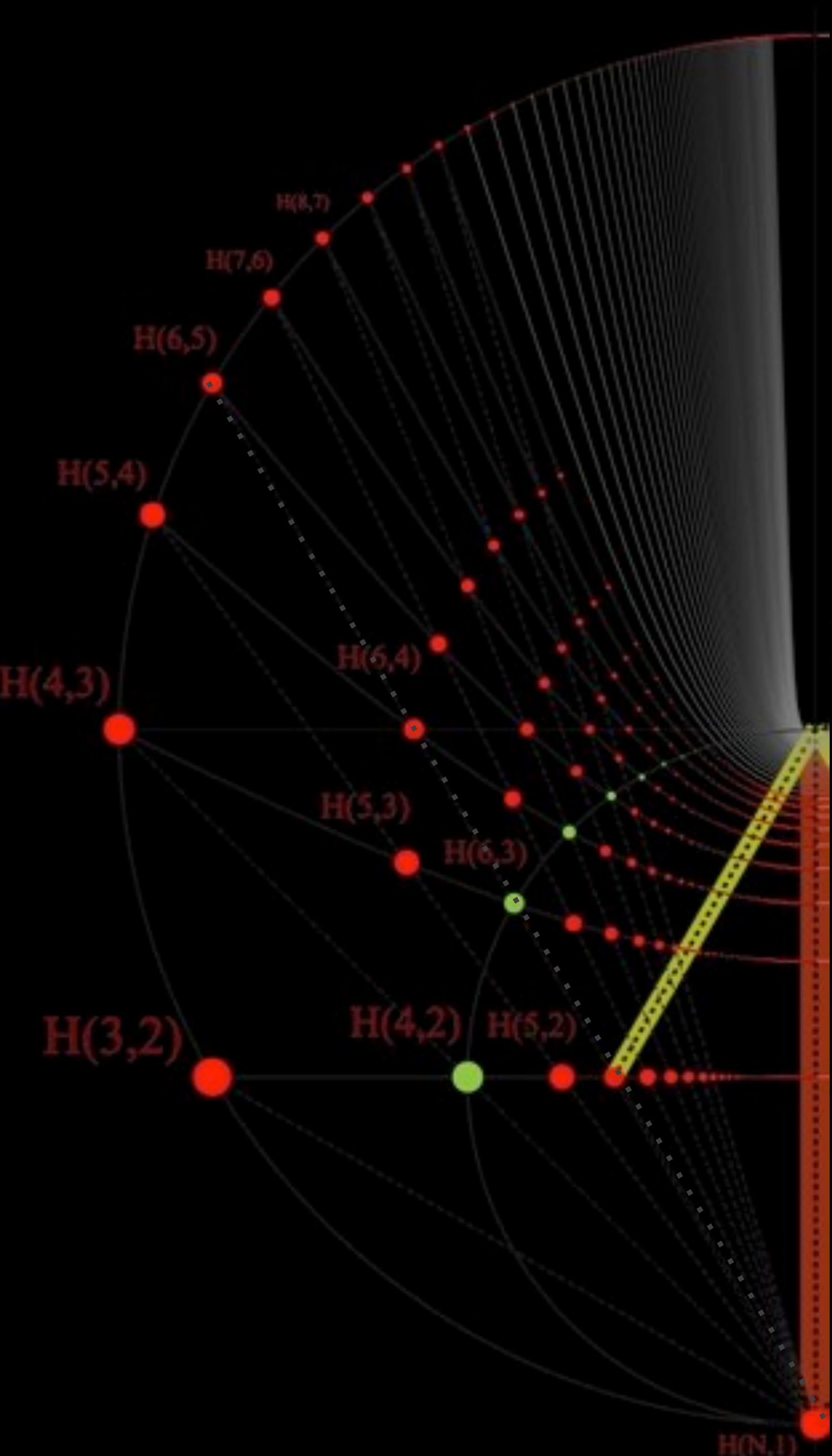
$$1/\Phi = \Phi - 1 = (1+\sqrt{5})½ - 1 \approx 0.618\ldots$$

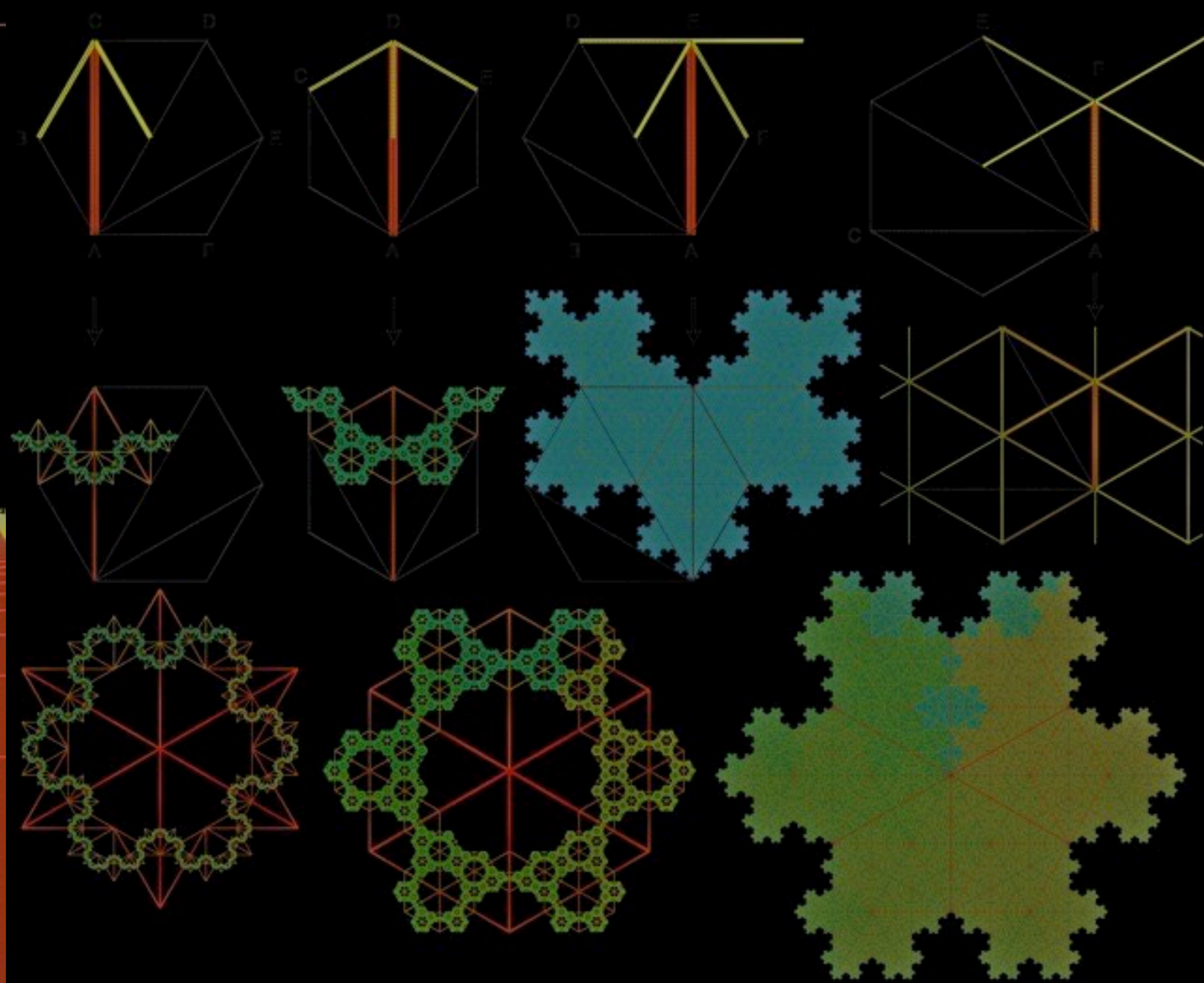
$$\Phi = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cdots}}} = \sqrt{1 + \sqrt{1 + \sqrt{1 + \cdots}}}$$

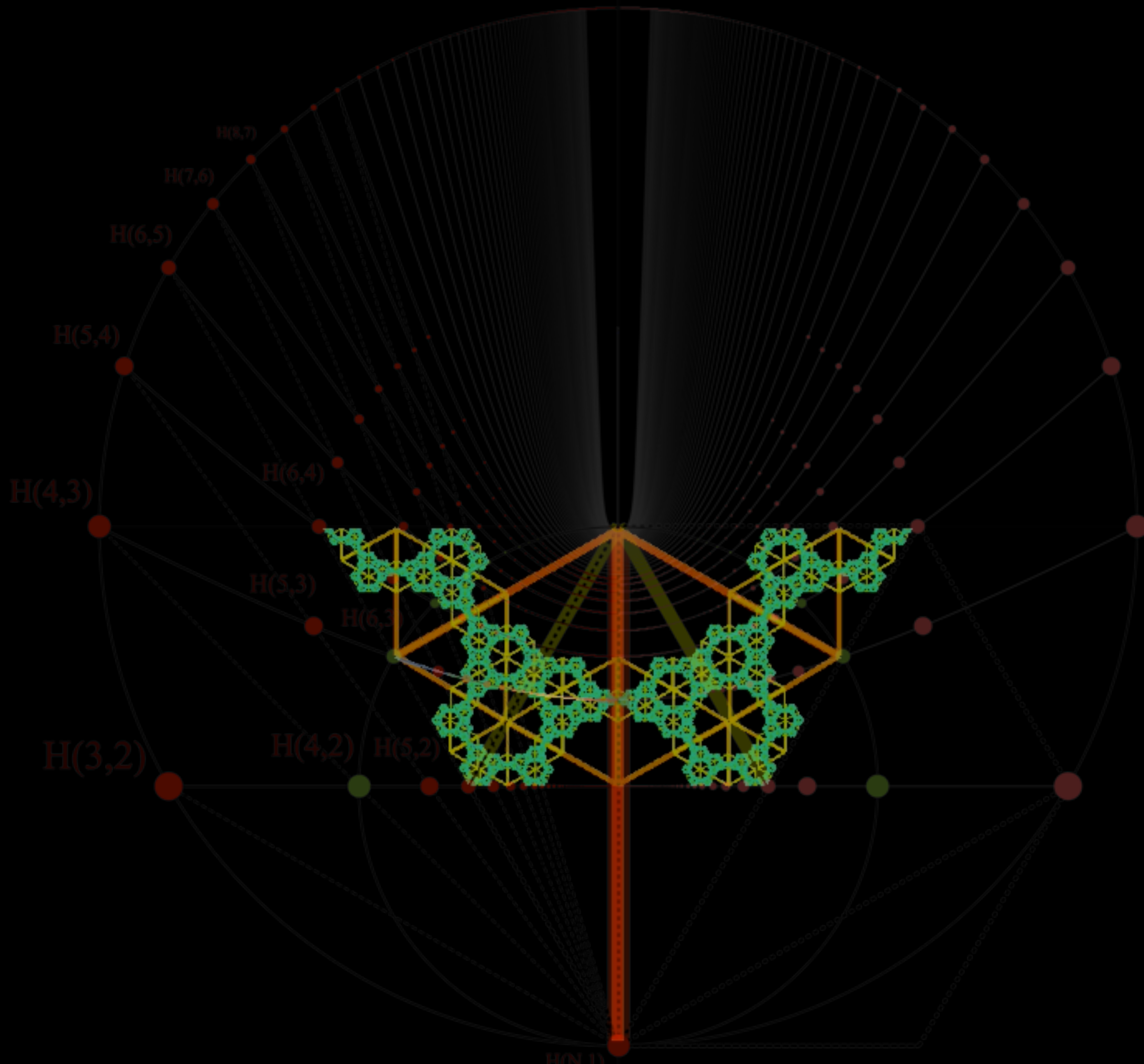


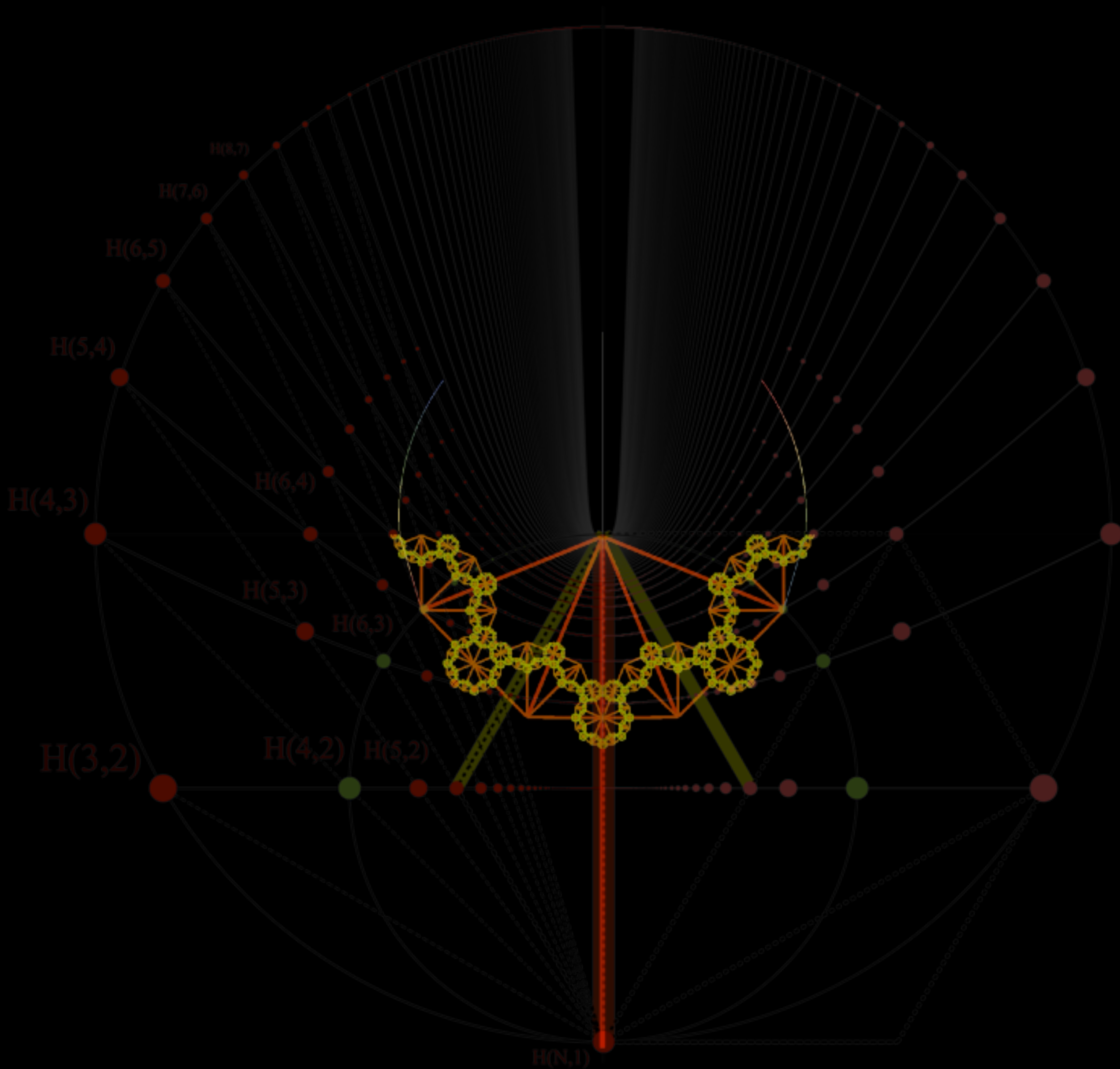

Silver Ratio
Golden Ratio
1
$\sqrt{2}$
1
ϕ
1
$\sqrt{3}$
2
4
5
6
7
8
9
10
∞
Angle θ

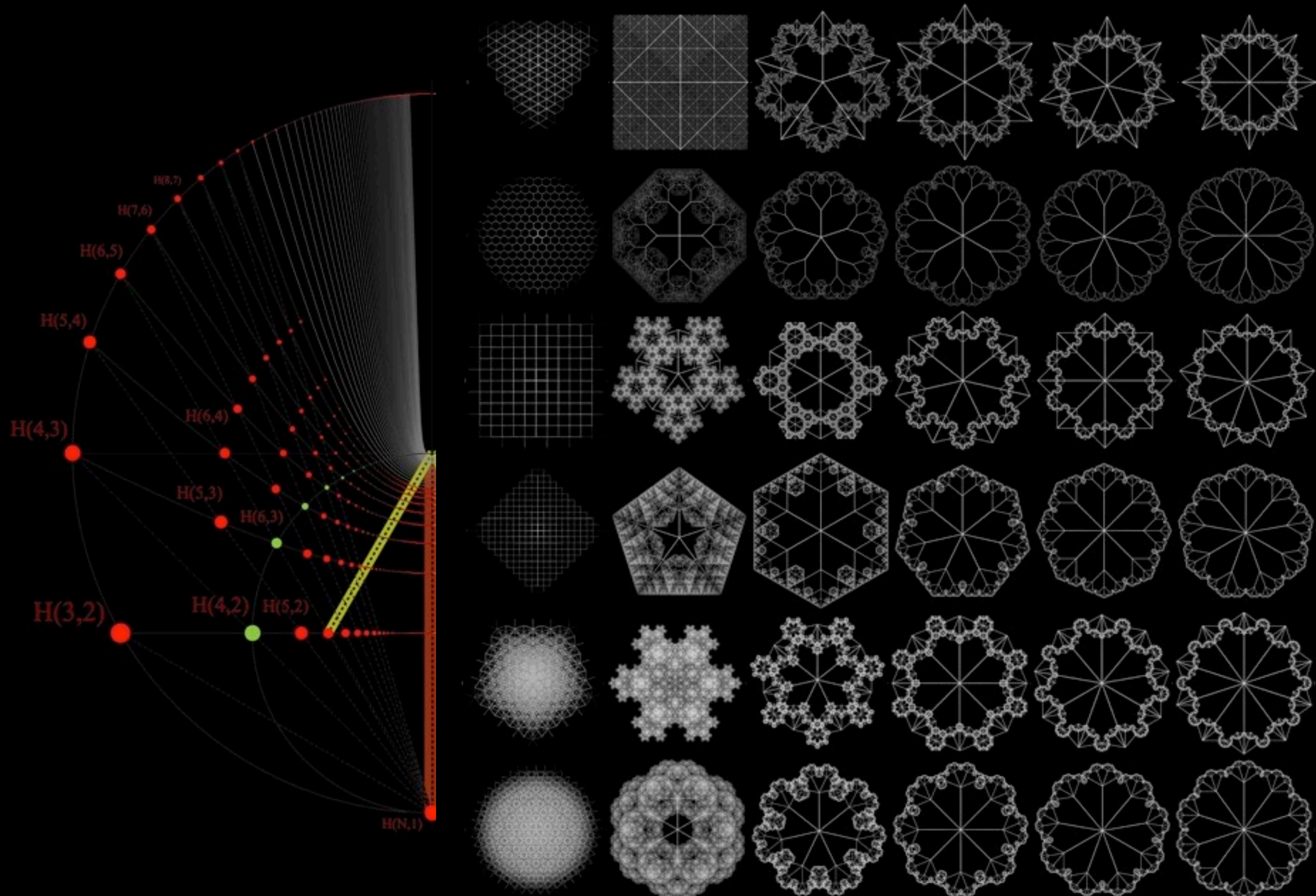
Harmonic Fractal Trees



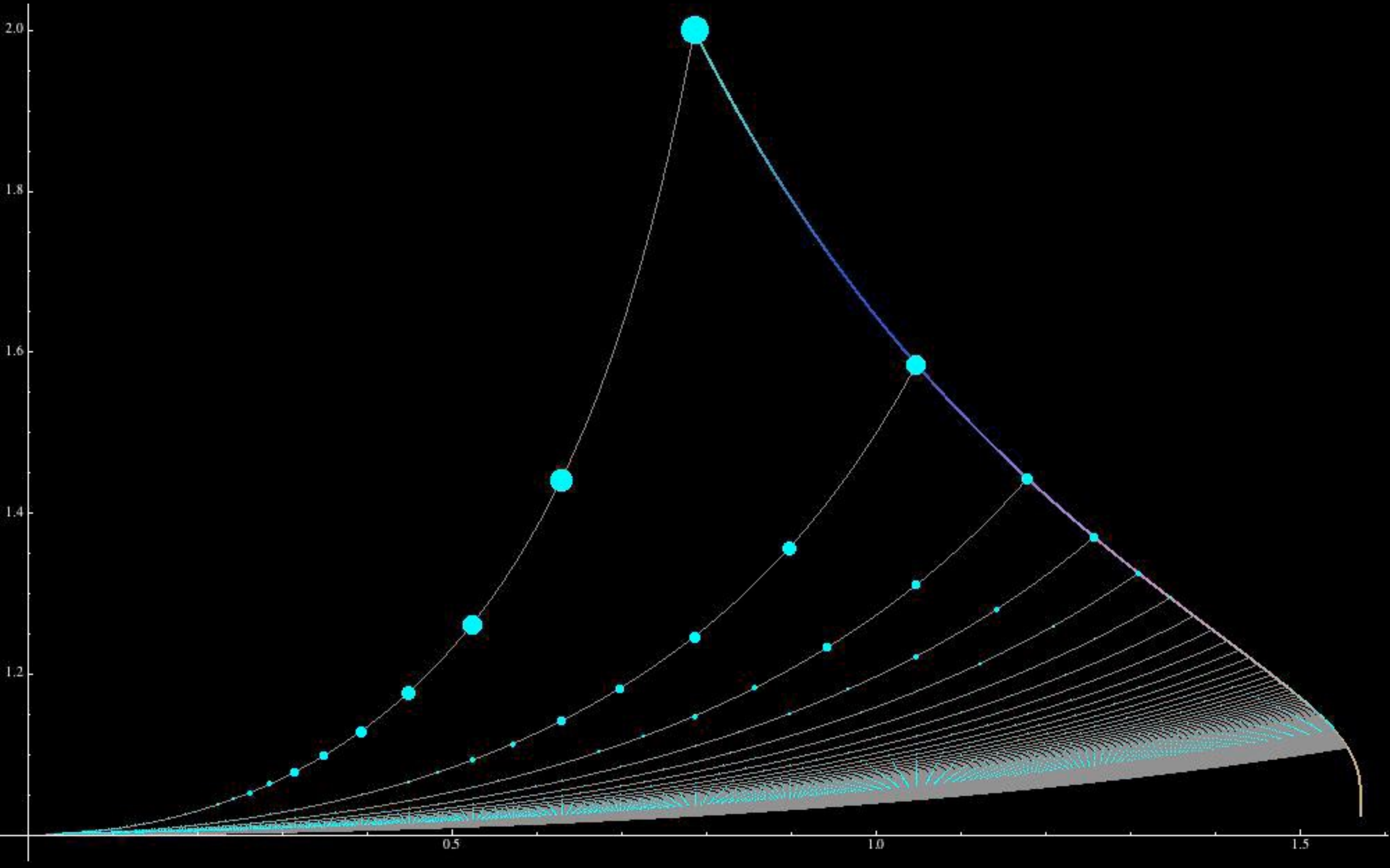


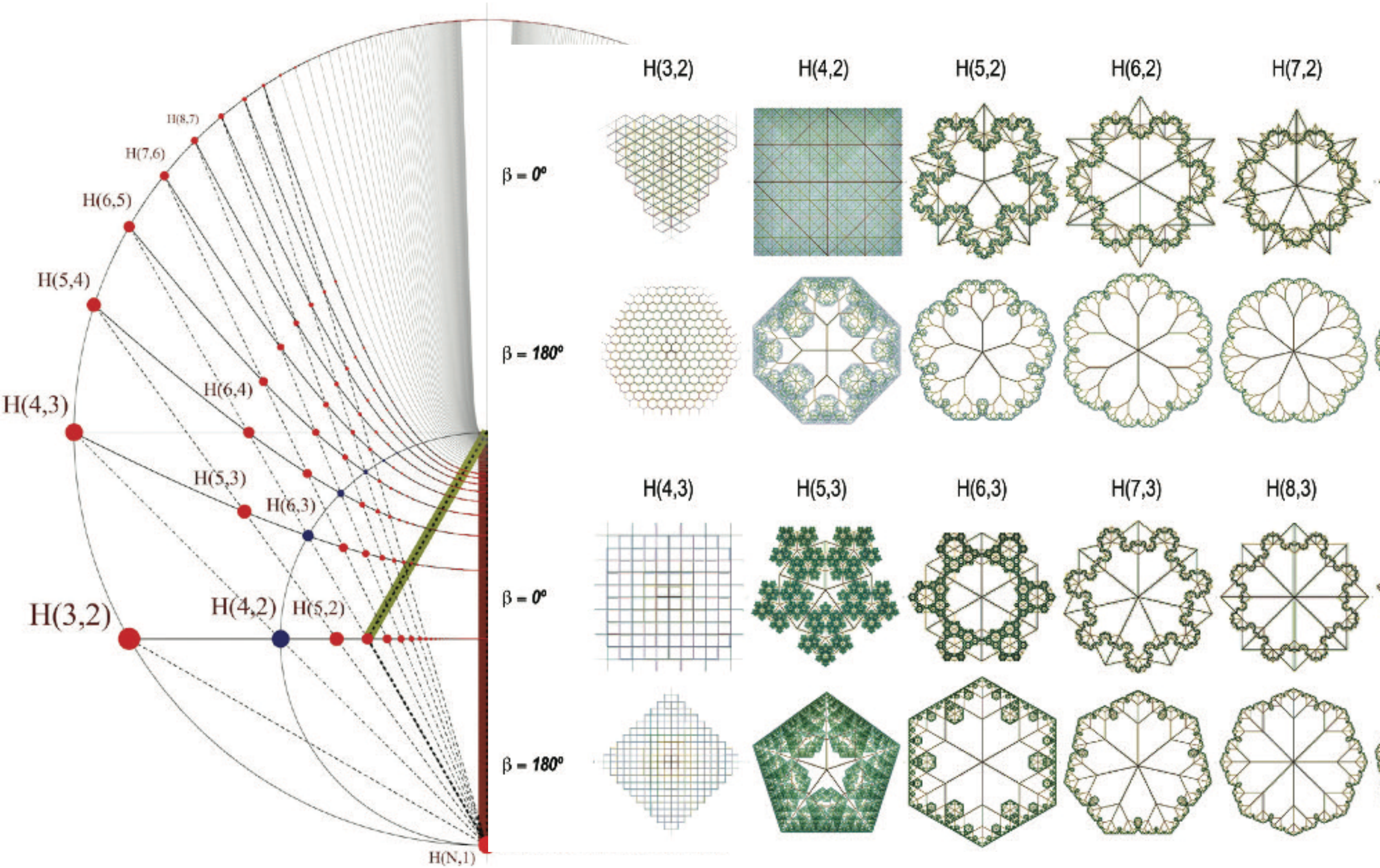

H(8,7)
H(7,6)
H(6,5)
H(5,4)
H(4,3)
H(6,4)
H(5,3)
H(6,3)
H(3,2)
H(4,2)
H(5,2)
H(N,1)


H(8,7)
H(7,6)
H(6,5)
H(5,4)
H(6,4)
H(4,3)
H(5,3)
H(6,3)
H(3,2)
H(4,2)
H(5,2)
H(N,1)



www.ComplexTrees.com


2.0
1.8
1.6
1.4
1.2
0.5
1.0
1.5


H(8,7)
H(7,6)
H(6,5)
H(5,4)
H(4,3)
H(6,4)
H(5,3)
H(6,3)
H(3,2)
H(4,2)
H(5,2)
H(N,1)
H(3,2)
H(4,2)
H(5,2)
H(6,2)
H(7,2)
β = 0°
β = 180°
H(4,3)
H(5,3)
H(6,3)
H(7,3)
H(8,3)
β = 0°
β = 180°

www.ComplexTrees.com

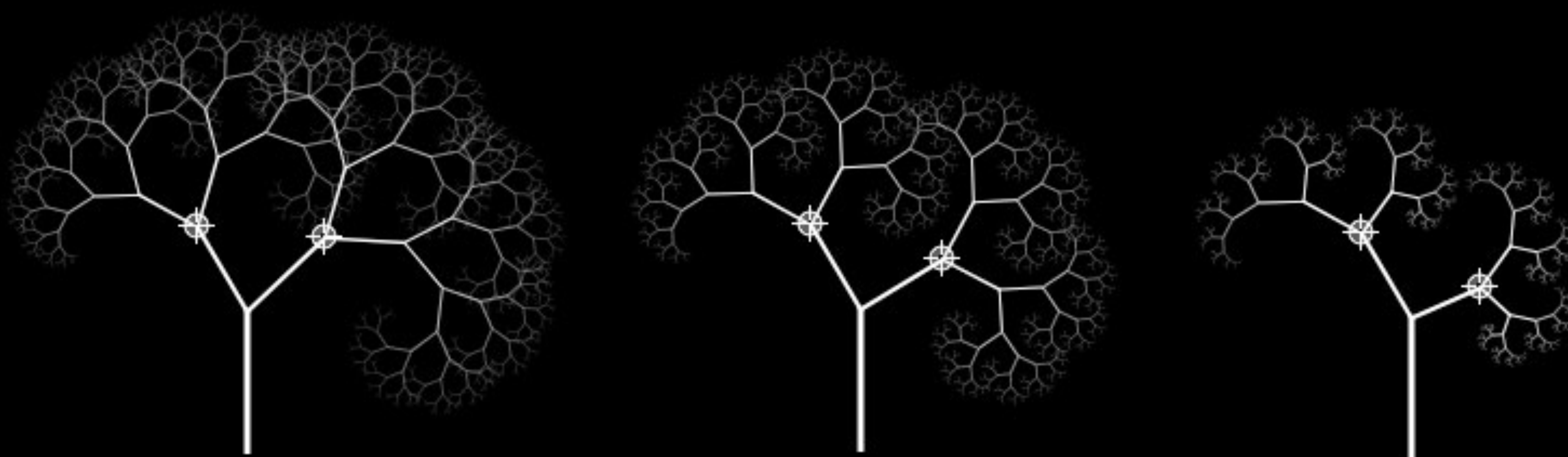
Thank you!

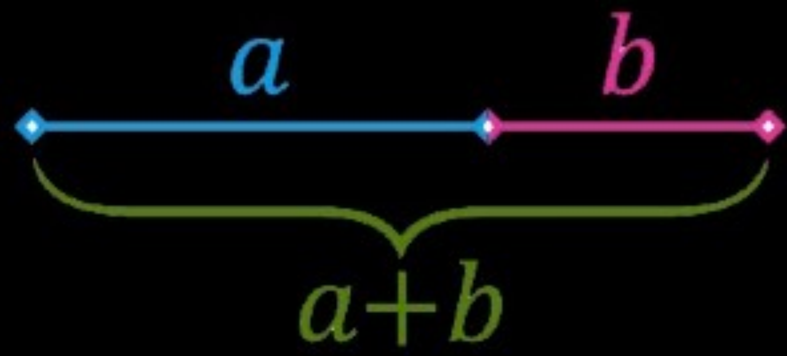
"If you wish to advance into the infinite, explore the finite in all directions."

- Goethe.

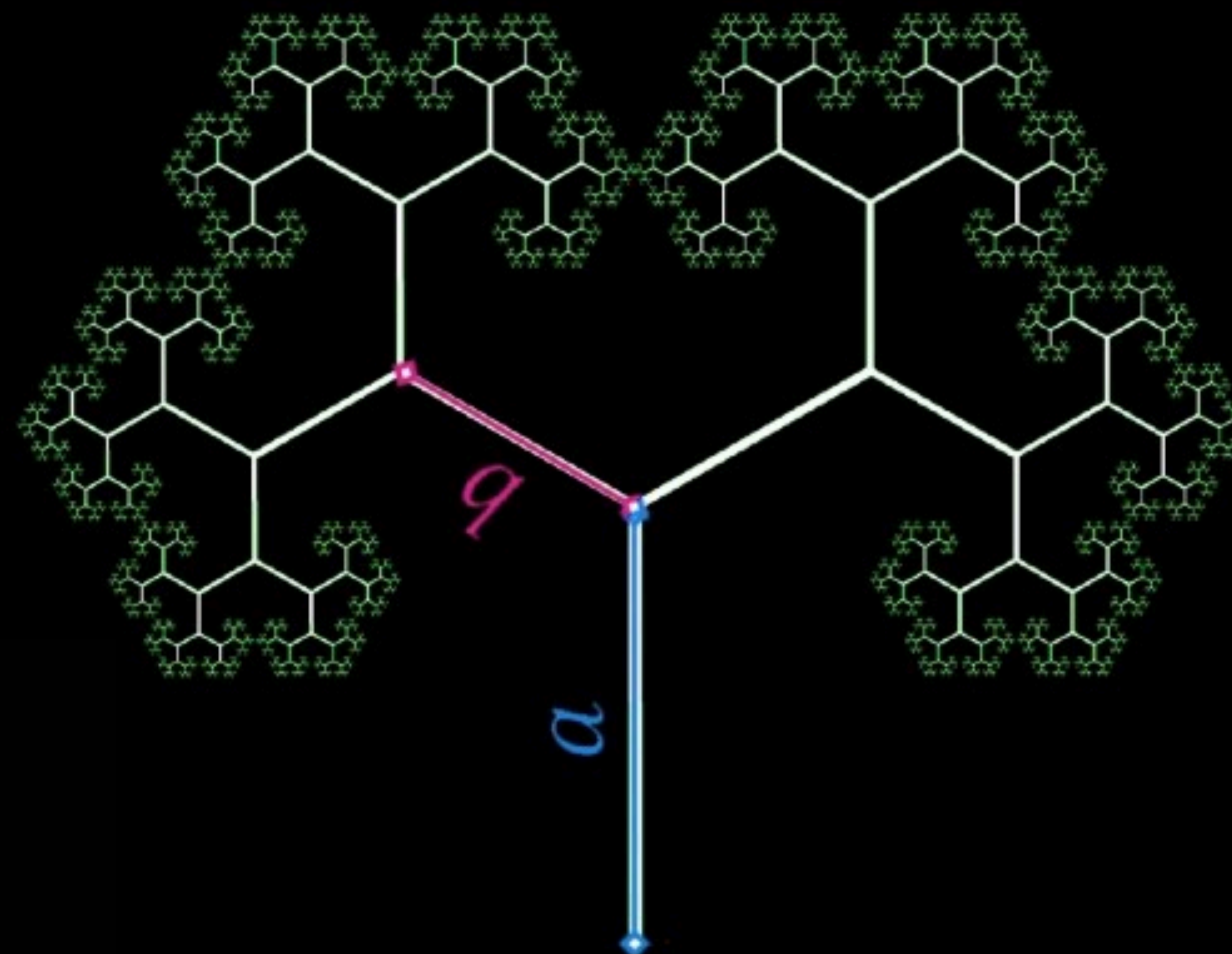
Bernat Espigulé

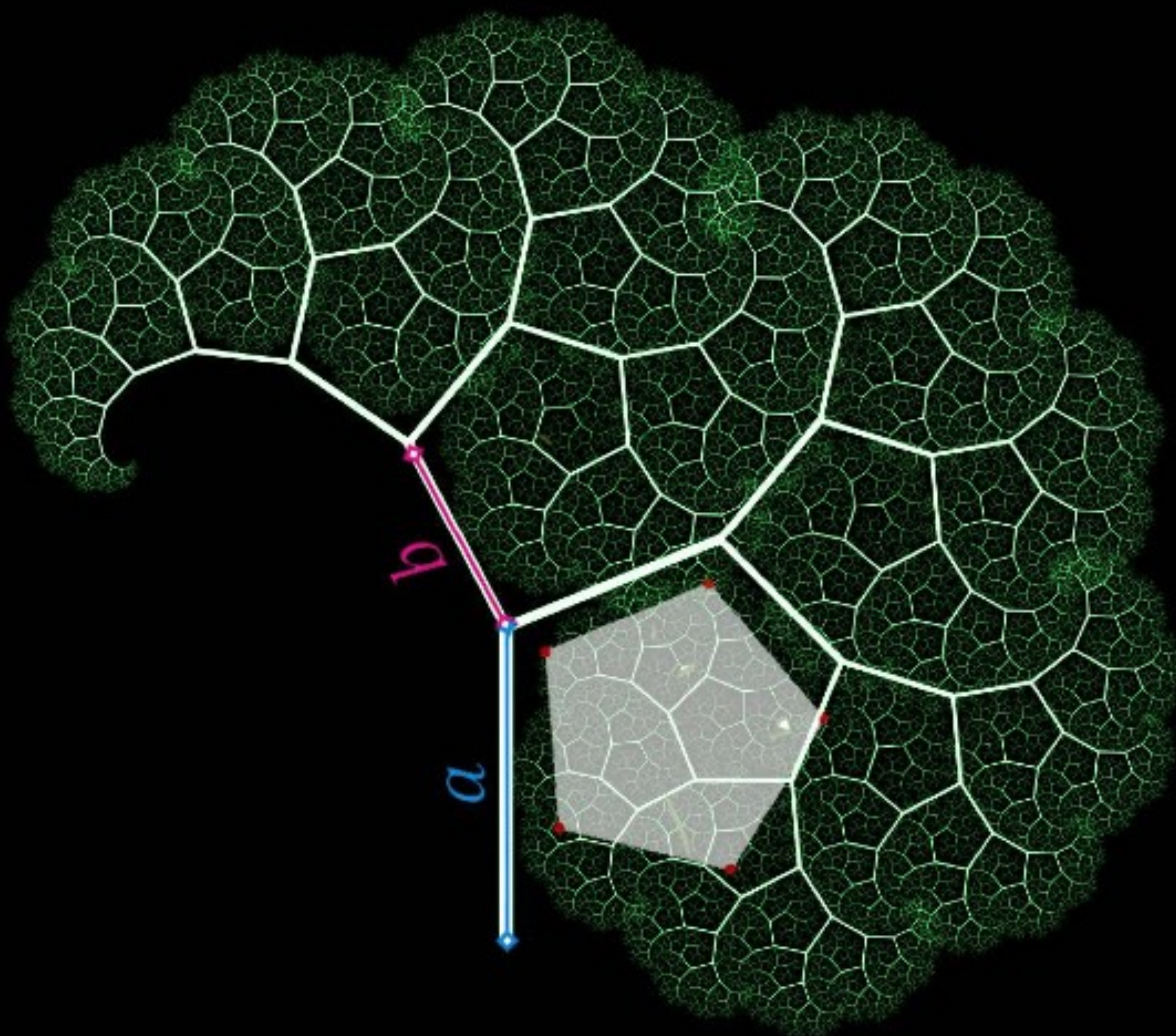
Holomorphic Dynamics Group
Núria Fagella & Xavier Jarque

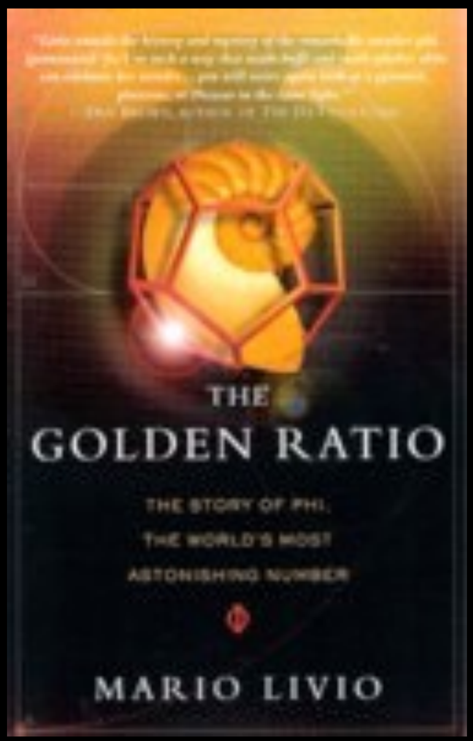


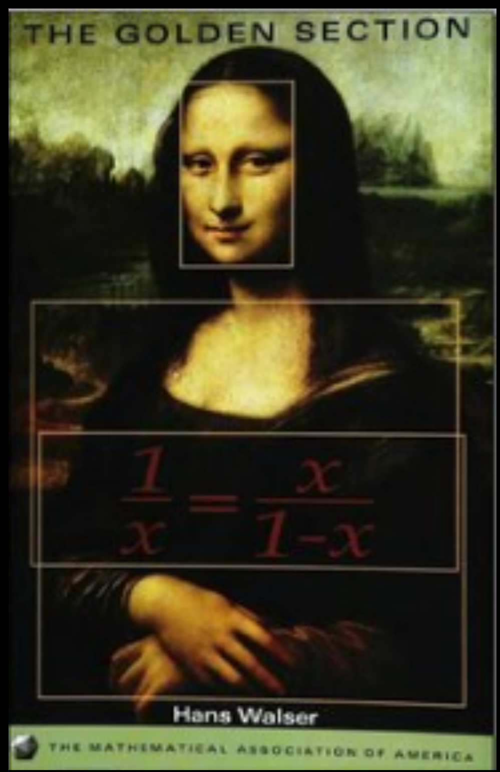


$$\frac{a+b}{a} = \frac{a}{b} \equiv \varphi = \frac{1+\sqrt{5}}{2} = 1.61803\,39887\ldots$$



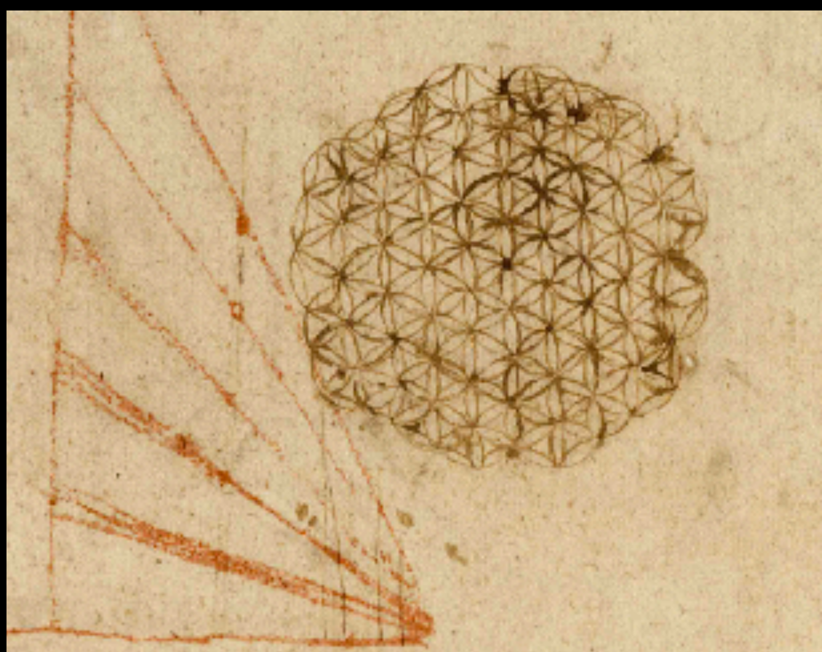


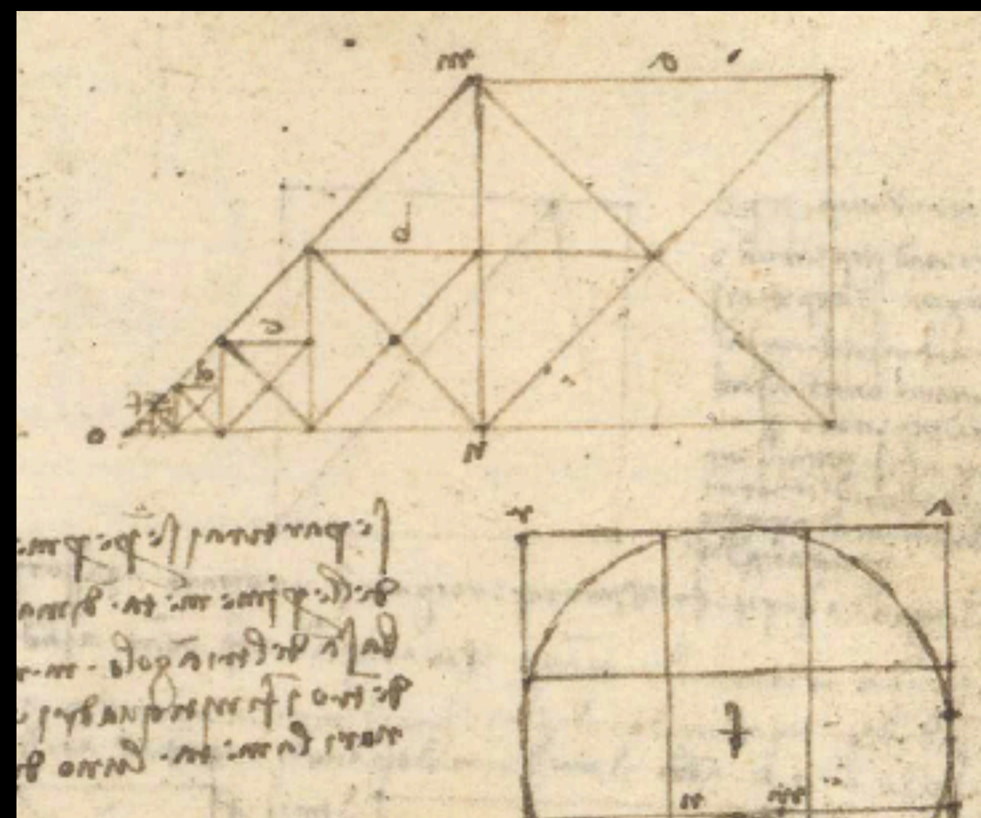


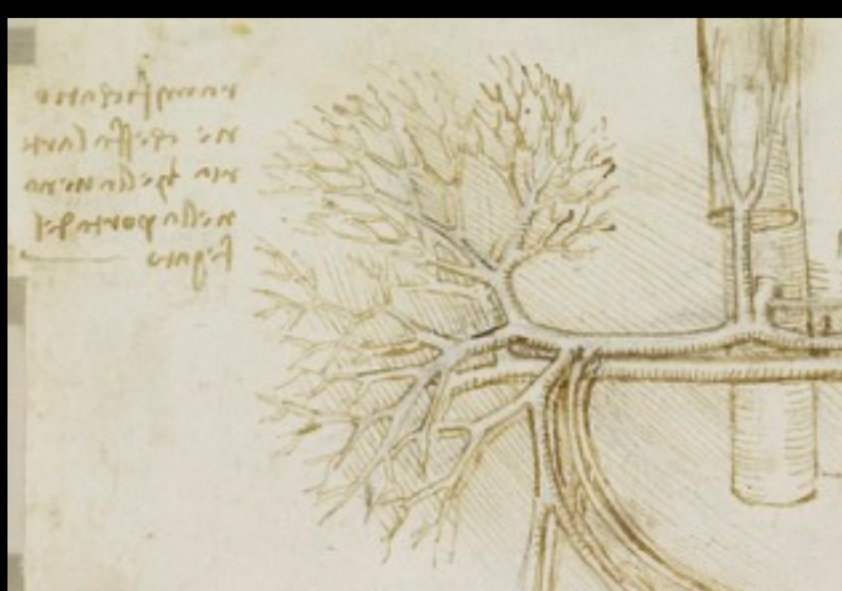


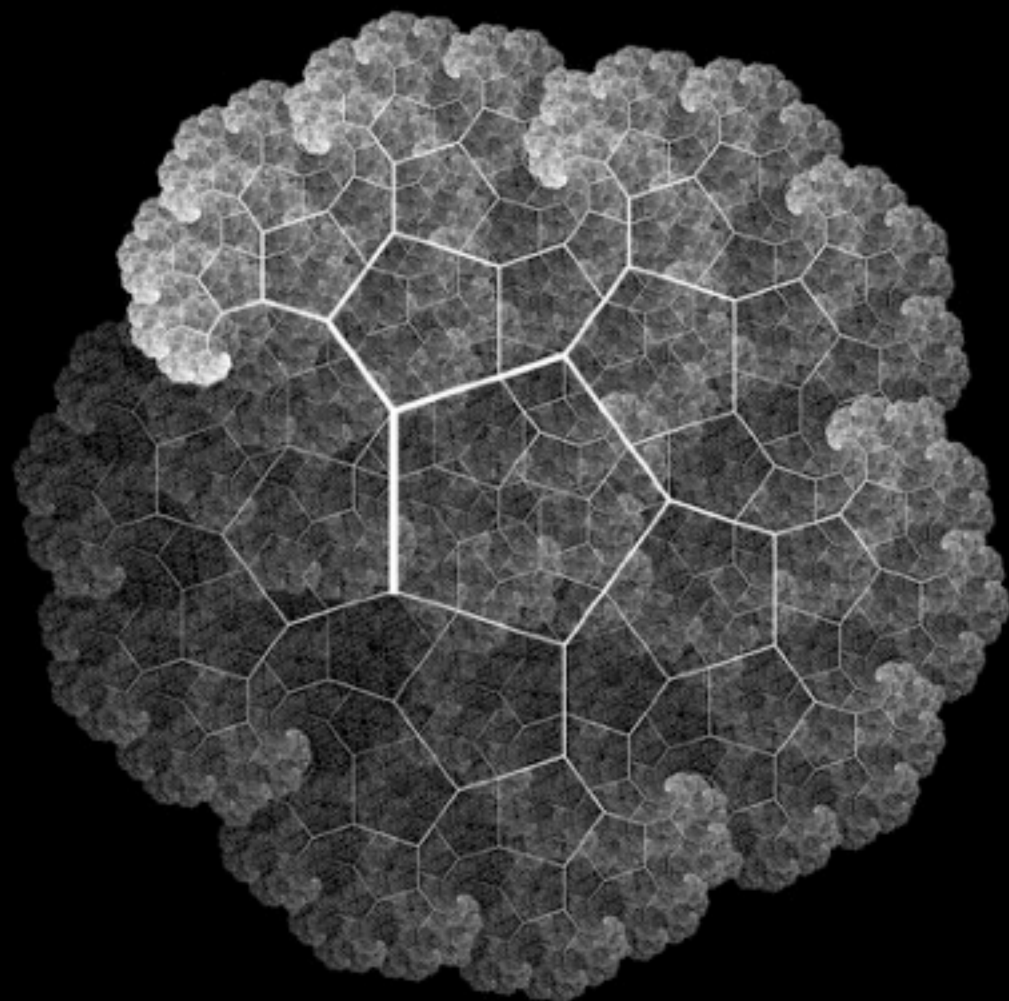
“Our soul is composed of harmony, and harmony is never bred save in moments when the proportions of objects are seen or heard.”

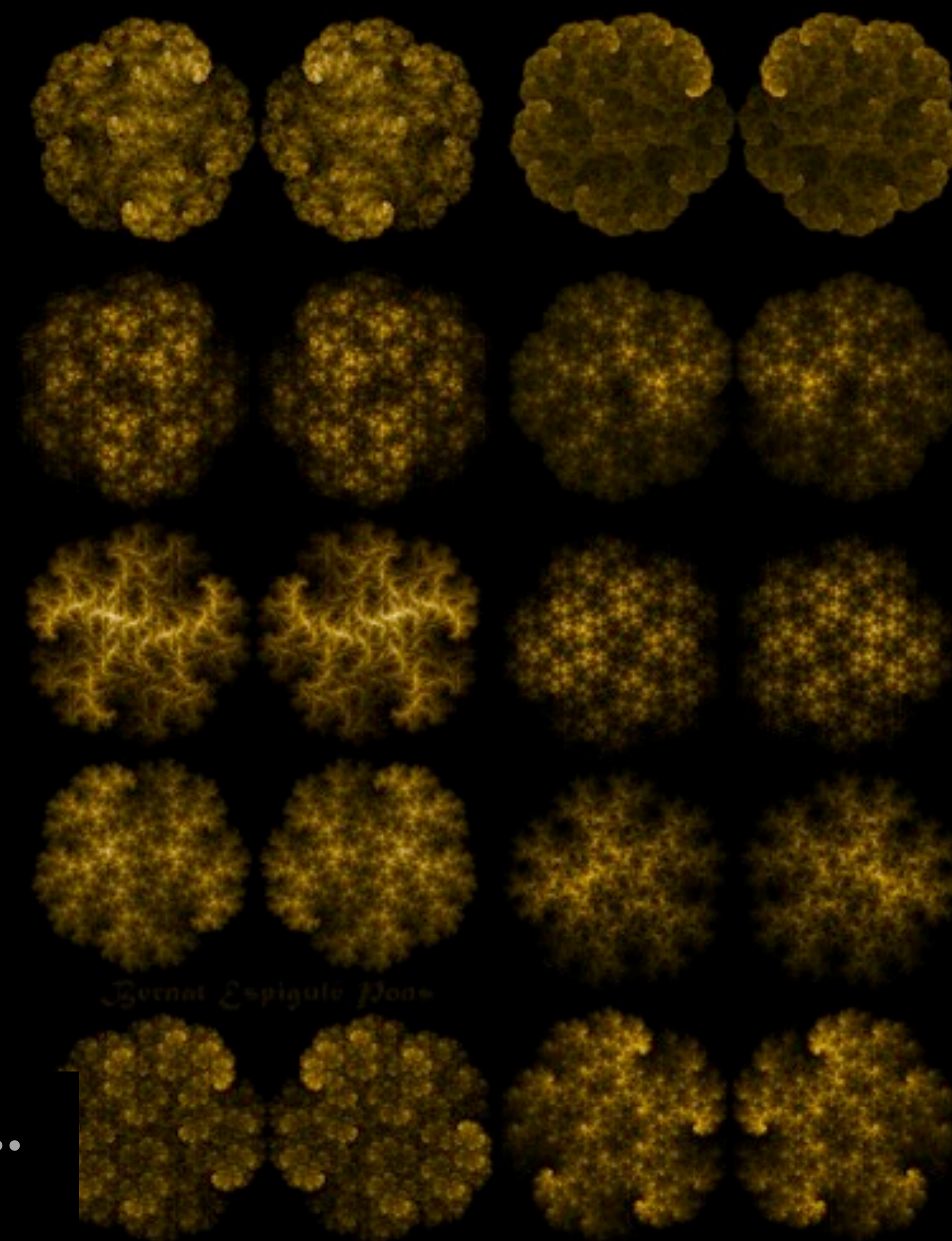
\- Leonardo da Vinci



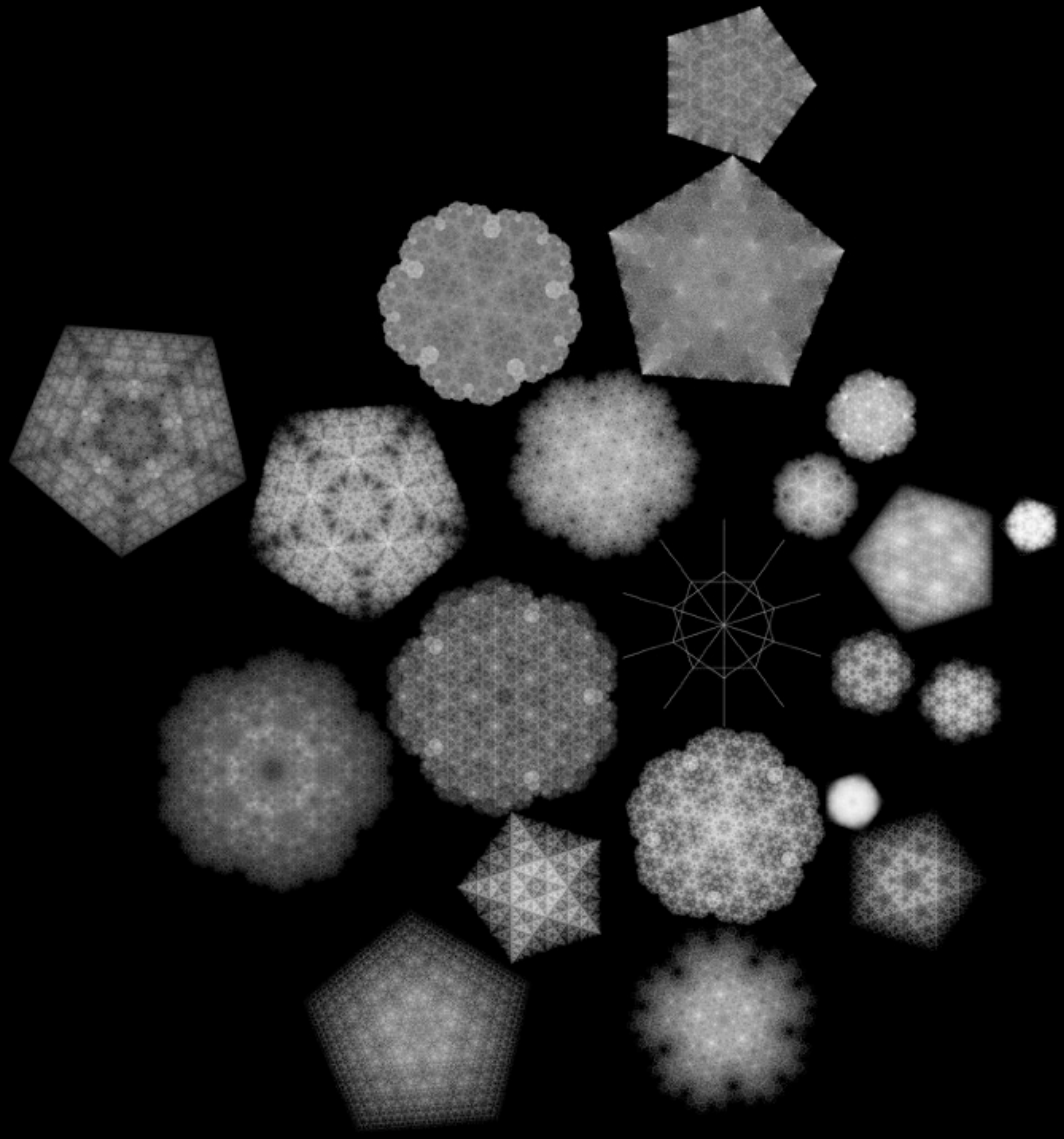


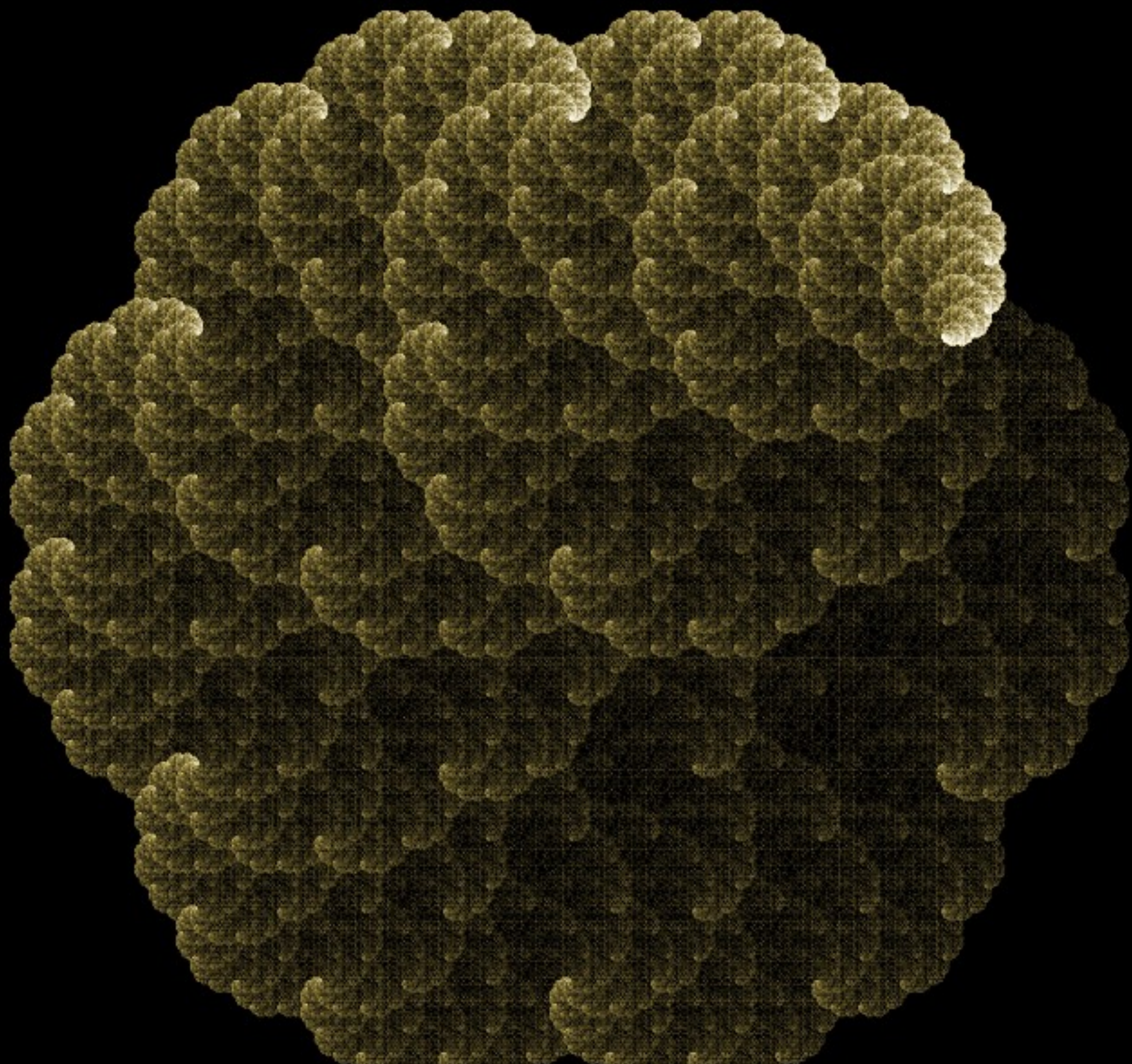


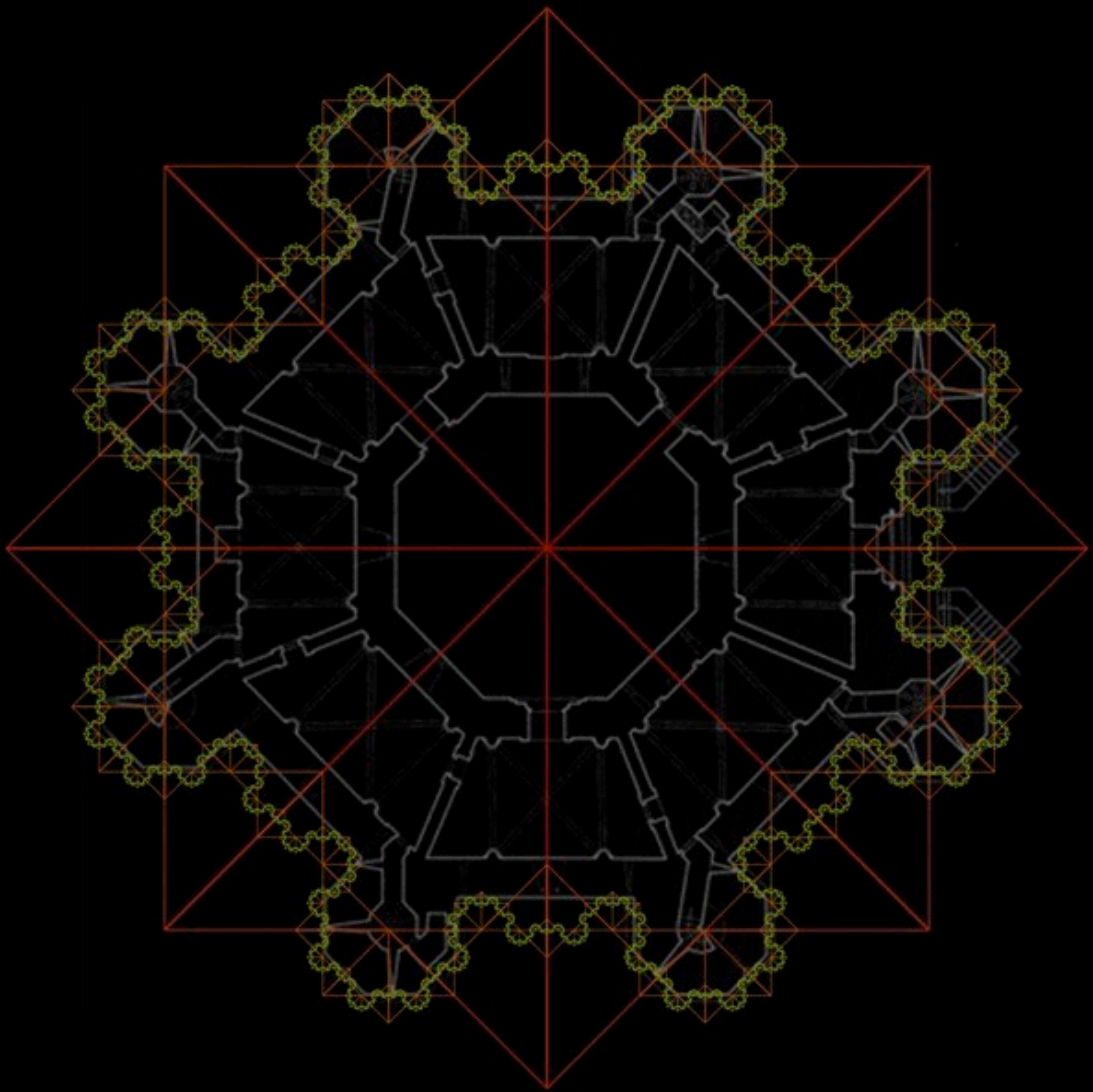


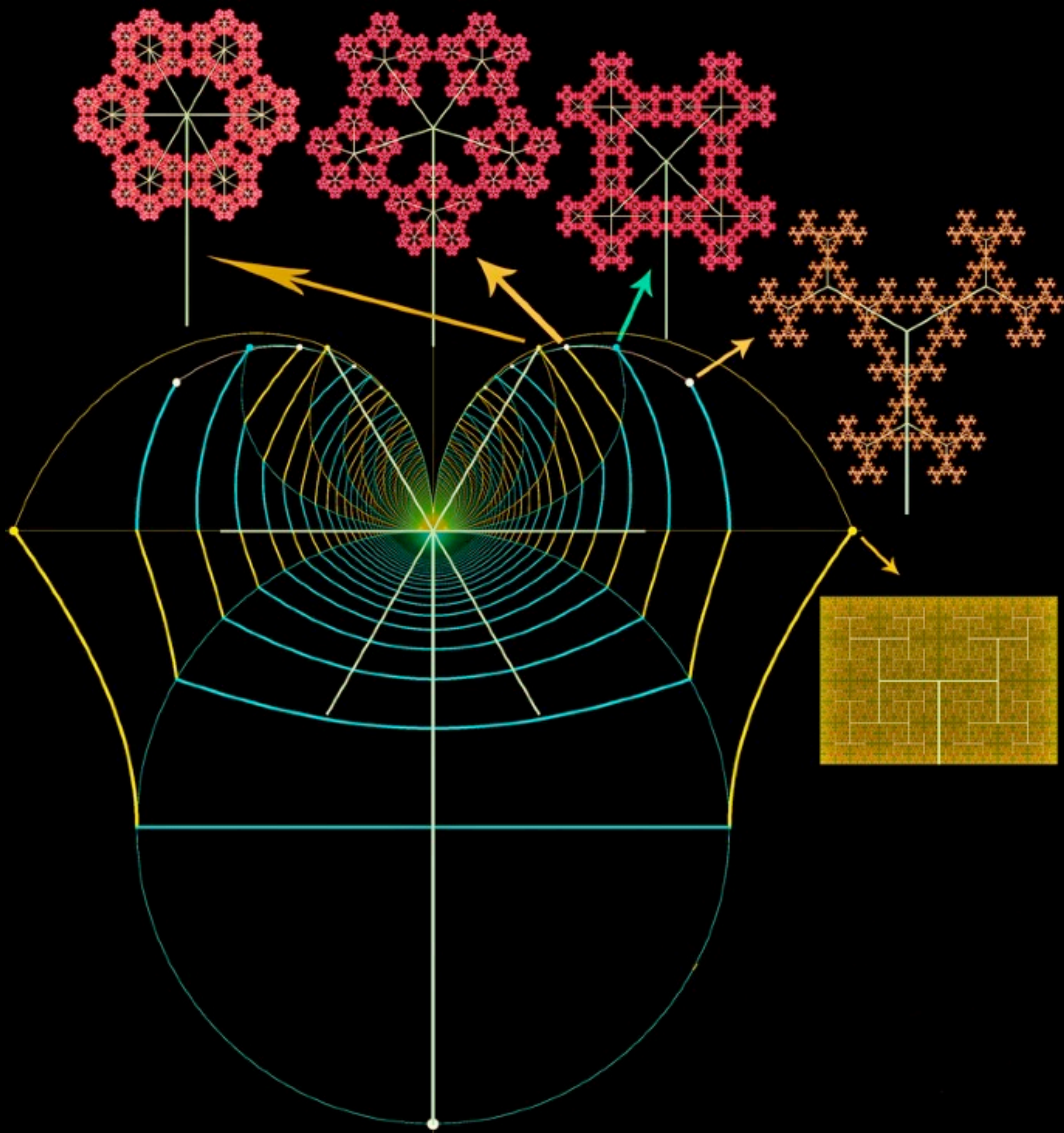


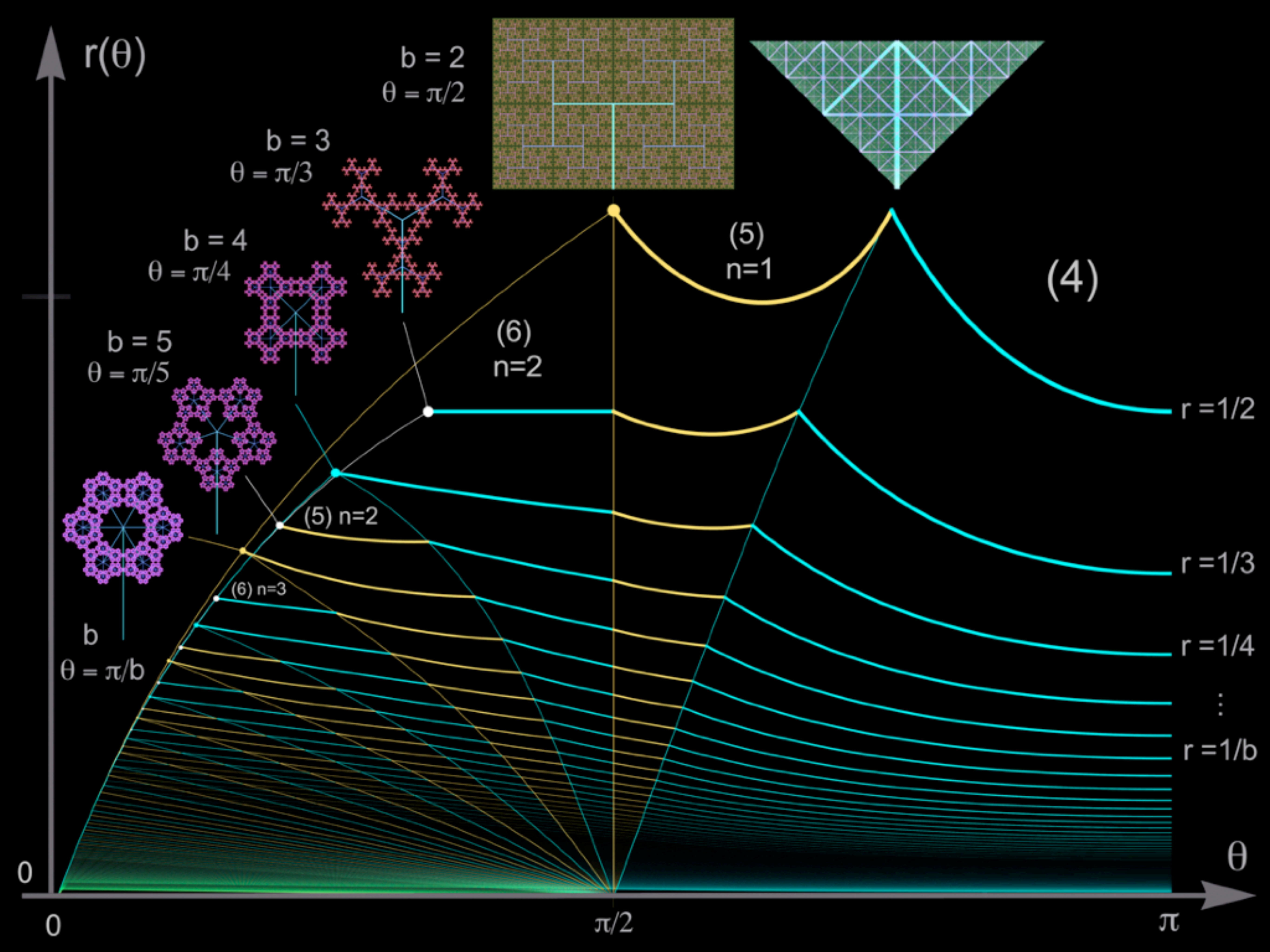
$$1/\Phi = \Phi - 1 = (1+\sqrt{5})\tfrac{1}{2} - 1 \approx 0.618\ldots$$

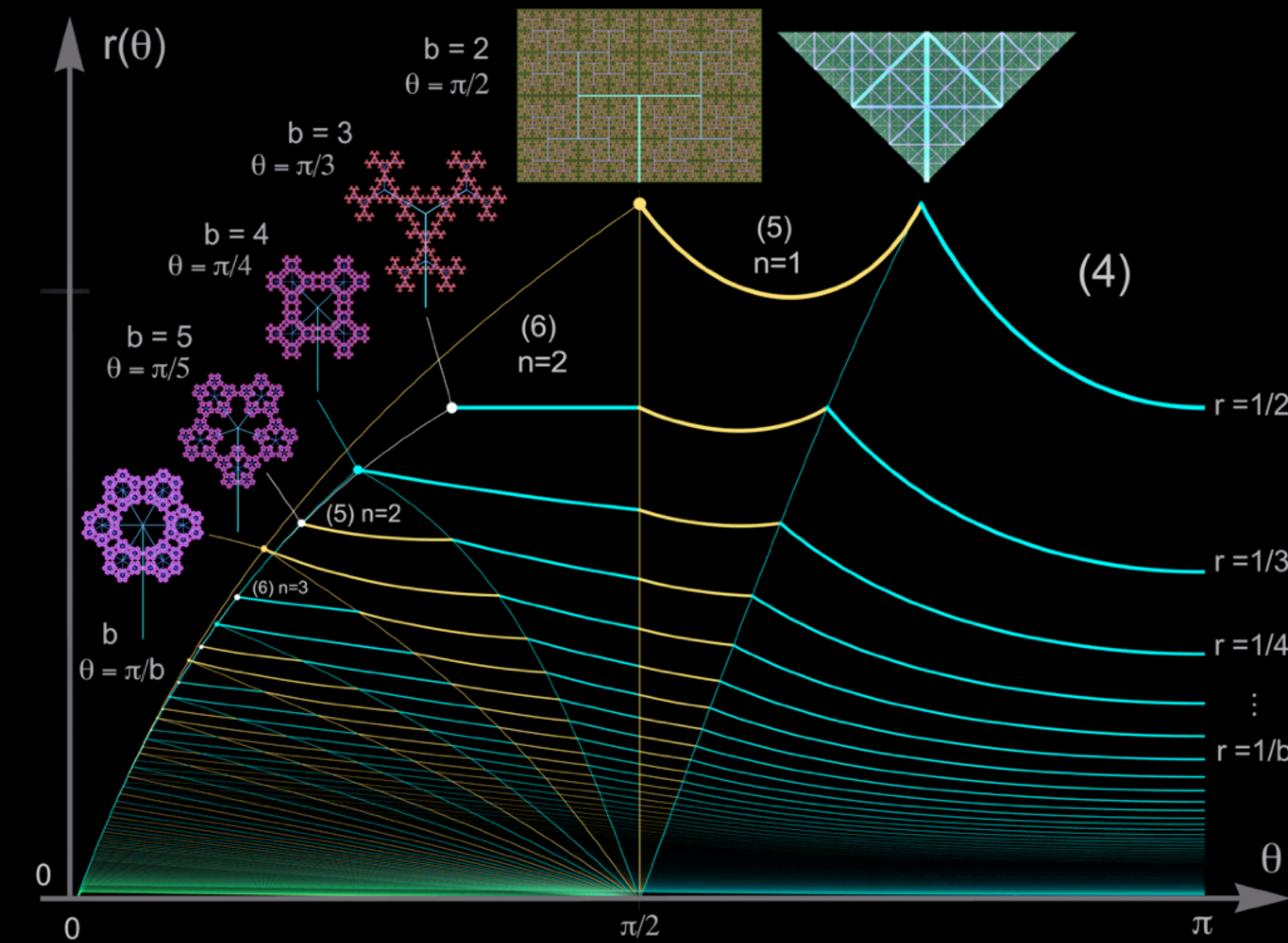









r(θ)
b = 2
θ = π/2
b = 3
θ = π/3
b = 4
θ = π/4
b = 5
θ = π/5
b
θ = π/b
(5)
n=1
(4)
(6)
n=2
(5) n=2
(6) n=3
r =1/2
r =1/3
r =1/4
r =1/b
θ
0
0
π/2
π



$$\frac{(b+1)\pi}{2b} \leq \theta \leq \pi$$

$$r_{sc} = -\sin\left(\frac{\pi-\theta}{b-1}\right)\csc\left(\frac{\pi-b\theta}{b-1}\right) \quad (4)$$

$$\frac{\pi}{b} \leq \theta < \frac{(b+1)\pi}{2b}$$

$$r = \frac{2\sin\left(\frac{\pi-\theta}{b-1}\right)}{\sqrt{\sin^2\left(\frac{\pi(3-2n)-\theta(b-2n+2)}{b-1}\right)+4\sin\left(\frac{\pi-\theta}{b-1}\right)\left(\sin\left(\frac{\pi-\theta}{b-1}\right)+\sin\left(\frac{(\pi-\theta)(2n-3)}{b-1}+2\theta\right)\right)}+\sin\left(\frac{\theta(b-2n+2)+\pi(2n-3)}{b-1}\right)}$$

$$\frac{\pi(b-4n+3)}{2(b-2n+1)} \leq \theta < \frac{\pi(b-4n+5)}{2(b-2n+2)} \qquad (5)$$

$$r_{sc} = \frac{\sqrt{\sin^2\left(\frac{\pi(3-2n)-\theta(b-2n+2)}{b-1}\right)+4\sin\left(\frac{\pi-\theta}{b-1}\right)\left(\sin\left(\frac{\pi-\theta}{b-1}\right)+\sin\left(\frac{(\pi-\theta)(2n-3)}{b-1}\right)\right)}+\sin\left(\frac{\pi(3-2n)-\theta(b-2n+2)}{b-1}\right)}{2\left(\sin\left(\frac{\pi-\theta}{b-1}\right)+\sin\left(\frac{(\pi-\theta)(2n-3)}{b-1}\right)\right)}$$

$$\frac{\pi(b-4n+5)}{2(b-2n+2)} \leq \theta < \frac{\pi(b-4n+7)}{2(b-2n+3)} \qquad (6)$$

$$L_{b/2}\, L_n\, (R_1\, L_1)^{\infty}$$

$$r = \frac{\sqrt{\sin^2\left(\frac{\pi(3-2n)-\theta(b-2n+2)}{b-1}\right) + 4\sin\left(\frac{\pi-\theta}{b-1}\right)\left(\sin\left(\frac{\pi-\theta}{b-1}\right) + \sin\left(\frac{(\pi-\theta)(2n-3)}{b-1}\right)\right)} + \sin\left(\frac{\pi(3-2n)-\theta(b-2n+2)}{b-1}\right)}{2\left(\sin\left(\frac{\pi-\theta}{b-1}\right) + \sin\left(\frac{(\pi-\theta)(2n-3)}{b-1}\right)\right)}$$

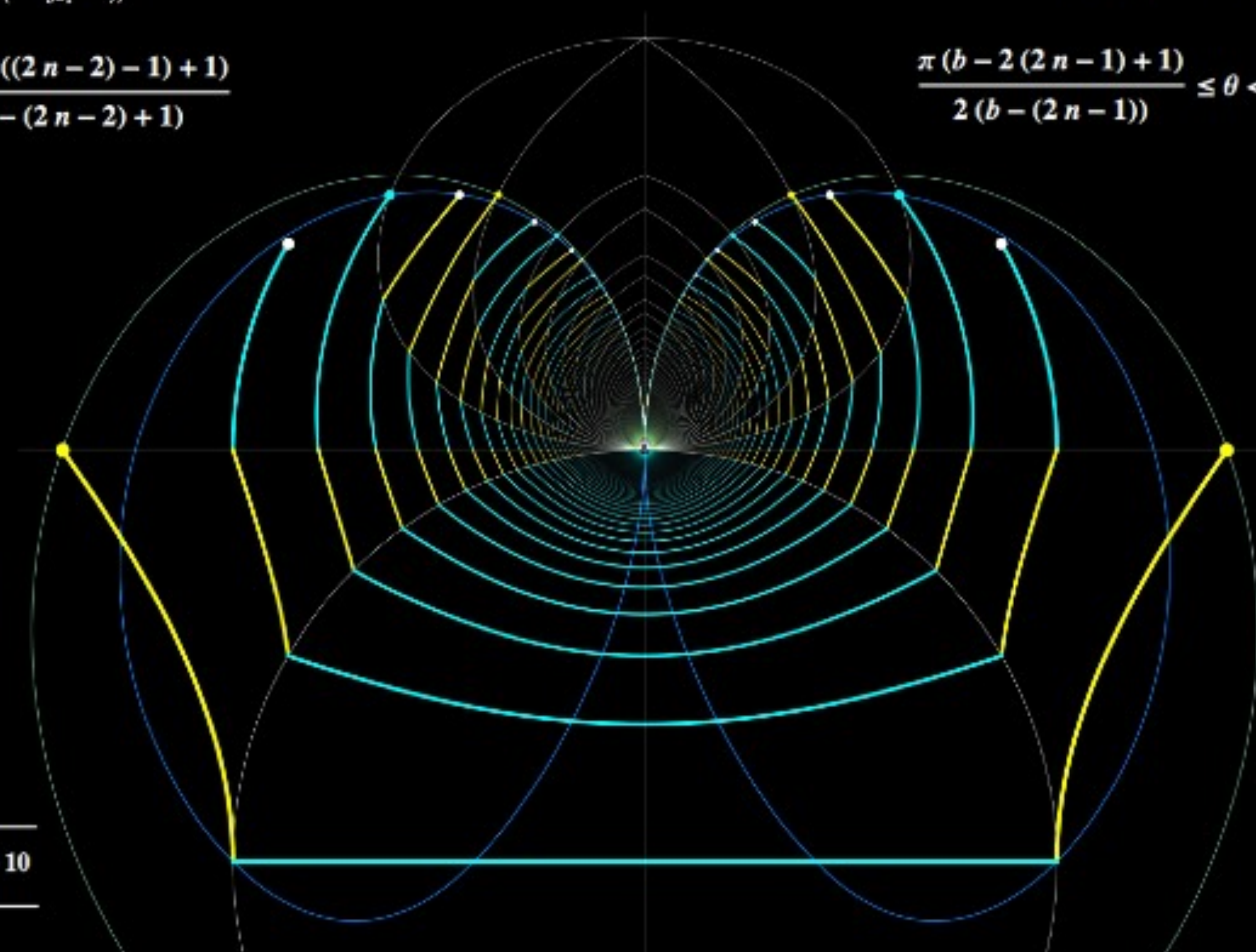
$$\frac{\pi(b-2(2n-2)+1)}{2(b-(2n-2))} \le \theta < \frac{\pi(b-2((2n-2)-1)+1)}{2(b-(2n-2)+1)}$$

$$2 \le n$$

$$L_{b/2}\, L_n\, (L_1\, R_1)^{\infty}$$

$$r = \frac{\sqrt{\sin^2\left(\frac{\pi(3-2n)-\theta(b-2n+2)}{b-1}\right) + 4\sin\left(\frac{\pi-\theta}{b-1}\right)\left(\sin\left(\frac{\pi-\theta}{b-1}\right) + \sin\left(\frac{2b\theta+\theta-2\theta n+2\pi n-3\pi}{b-1}\right)\right)} + \sin\left(\frac{\pi(3-2n)-\theta(b-2n+2)}{b-1}\right)}{2\left(\sin\left(\frac{\pi-\theta}{b-1}\right) + \sin\left(\frac{2b\theta+\theta-2\theta n+2\pi n-3\pi}{b-1}\right)\right)}$$

$$\frac{\pi(b-2(2n-1)+1)}{2(b-(2n-1))} \le \theta < \frac{\pi(b-2((2n-1)-1)+1)}{2(b-(2n-1)+1)}$$



$$L_{b/2}\, L_1\, (L_1\, R_1)^{\infty}$$

$$r = \frac{2\sin\left(\frac{\pi-b\theta}{b-1}\right) + \sqrt{-8\cos\left(\frac{2(\pi-\theta)}{b-1}\right) + 6\cos\left(\frac{2(\pi-b\theta)}{b-1}\right) - 8\cos(2\theta) + 10}}{4\left(\sin\left(\frac{-2b\theta+\theta+\pi}{1-b}\right) + \sin\left(\frac{\pi-\theta}{b-1}\right)\right)}$$

$$\frac{\pi}{2} \le \theta < \frac{(b+1)}{2b}\pi$$

$$L_{b/2}\, (L_1\, R_1)^{\infty}$$

$$r = -\sin\left(\frac{\pi-\theta}{b-1}\right)\csc\left(\frac{\pi-b\theta}{b-1}\right)$$

$$\frac{(b+1)}{2b}\pi \le \theta \le \pi$$