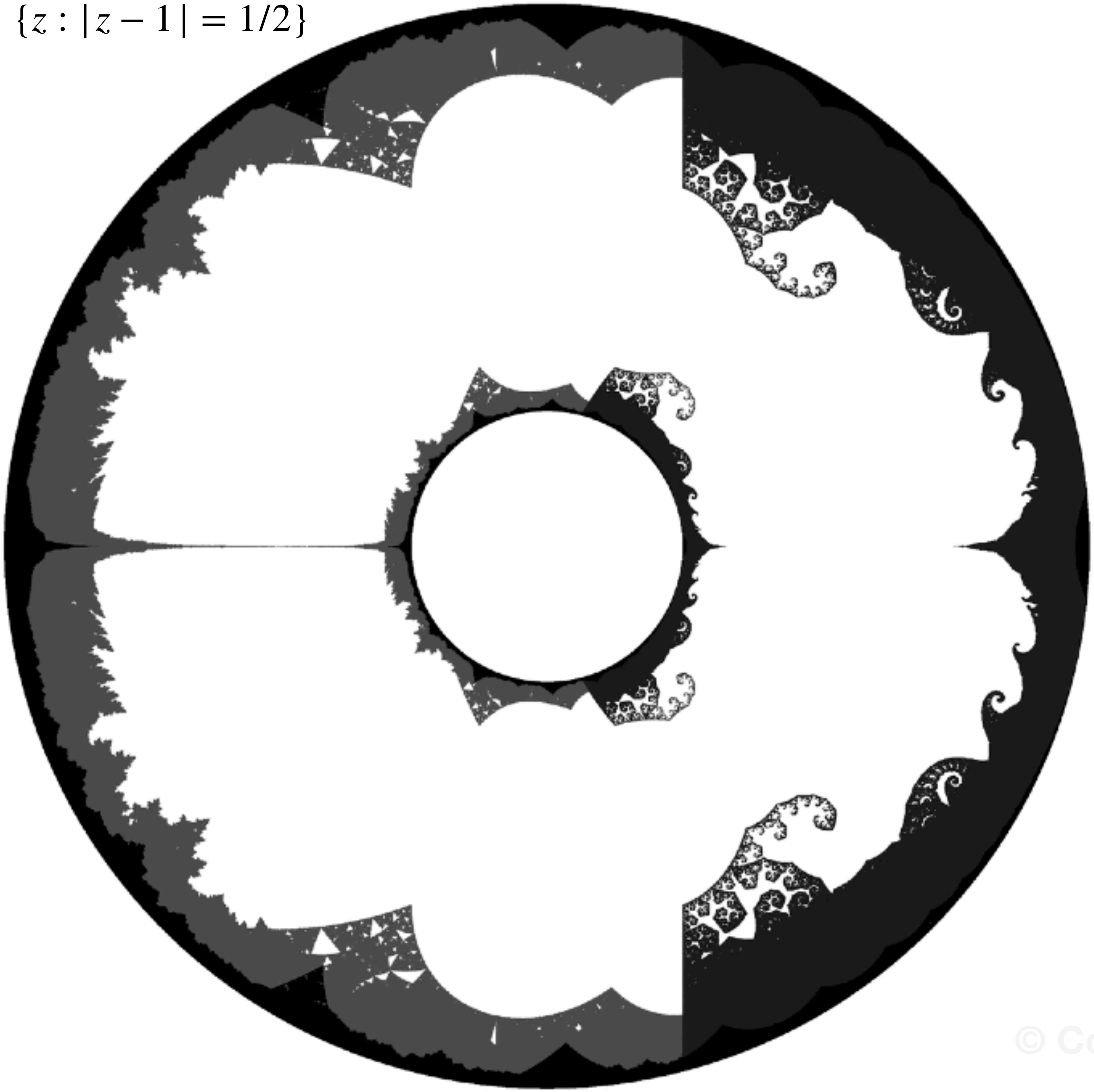
The Wolfram Set $W_\lambda := \{z : \lambda \in F_A(z)\}$ **where** $A(z) := \{z, 1/2, 1/4z\}$

$\lambda \in \{z : |z - 1| = 1/2\}$



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Thank you!

“If you wish to advance into the infinite, explore the finite in all directions.”

\- Goethe.

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